SOLUTIONS MANUAL FOR

Safeguard your electric power systems with . . .

PROTECTIVE RELAYING
PRINCIPLES AND APPLICATIONS

J. LEWIS BLACKBURN
Consultant, Bothell, Washington

MARCEL DEKKER, INC.

Introduction to the Problem Section:

The problems are intended to emphasize, enhance, and provide opportunities to apply the material covered in the various text chapters. These are practical problems. Many have developed from actual power system and industrial problems, and experiences accumulated over the years. Others are selected to provide checks on understanding and practice on basic fundamentals and techniques. Each problem and part has a "message" hopefully "unlocked" and understood with a reasonable minimum of "labor"; that is with a good RH factor (R = relative minimum labor, H = high educational value).

Problems involving application choices have been avoided as much as possible. In the relaying art these types tend to have many different solutions depending on personality, local conditions, practices, objectives, economics, and so forth.

The first number of each problem refers to the particular text chapter involved.

J. Lewis Blackburn
August 20, 1987

Protective Relaying: Principles & Applications.

J Lewis Blackburn

Published by Marcel Dekker, Inc. New York 1987.

Problem Solutions & Answers: Pages 499-532.

Prob. 2.1 pg 499:

$$\text{ohms} = \frac{kV^2 Z_{pu}}{MVA} \quad \text{eq. 2.17}$$

$$= \frac{20^2 \times 1.2}{200} = 2.40 \text{ ohms at } 20\,kV.$$

Prob 2.2 pg 499:

$$Z_{2pu} = Z_{1pu} \frac{MVA_2}{MVA_1} \left(\frac{kV_1}{kV_2}\right)^2 \quad \text{eq. 2.33}$$

$$= 1.2 \times \frac{100}{200} \left(\frac{20}{13.8}\right)^2 = 1.26 \text{ pu at } 100\,MVA, 13.8\,k$$

Prob 2.3 pg 499:

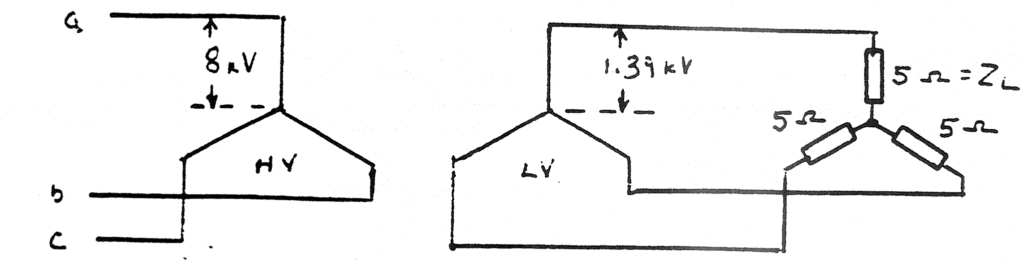
$$\text{ohms} = \frac{13.8^2 \times 1.26}{100} = 2.40 \text{ ohms at } 13.8\,kV.$$

yes - same value as Prob. 2.1.

Prob. 2.4 pg 499-450:

Case No.	Transformer Connection		Load Connection to Sec.	Line-to-line Base kV		Load in Per Unit	Total Z as Viewed From The High Side	
	Pri.	Sec.		HV	LV	R	Per Unit	Ohms
1	wye	wye	wye	13.8	2.4	13.0	13+j.06	165.1+j.76
2	wye	wye	delta	13.8	2.4	4.34	4.34+j.06	55.1+j.76
3	wye	delta	wye	13.8	1.39	38.8	38.8+j.06	493+j.76
4	wye	delta	delta	13.8	1.39	13.0	13+j.06	165.1+j.76
5	delta	wye	wye	8	2.4	13.0	13+j.06	55.2+j.256
6	delta	wye	delta	8	2.4	4.34	4.34+j.06	18.6+j.256
7	delta	delta	wye	8	1.39	38.8	38.8+j.06	165.6+j.256
8	delta	delta	delta	8	1.39	13.0	13+j.06	55.2+.256

Prob 2.4 Cont'd.



$X_T = 6\%$

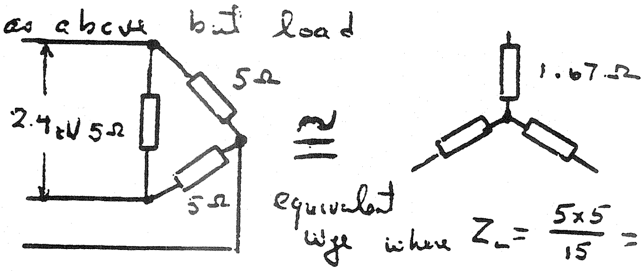
Case 1. HV Base = $8\sqrt{3} = 13.8$ kV ; LV Base $= 1.39\sqrt{3} = 2.4$ kV.

eq 2.1 $\quad Z_{B_H} = \dfrac{13.8^2}{15} = 12.7$ ohms at 13.8 kV ; $Z_{B_L} = \dfrac{2.4^2}{15} = 0.384$ ohms at 2.4 kV

$Z_L = \dfrac{5}{0.384} = 13$ pu $\quad$ or eq 2.15, $Z_L = \dfrac{15 \times 5}{2.4^2} = 13$ pu. at 15 MVA

$Z_{Total} = 13 + j.06$ pu at 15 MVA, 13.8 kV $= \dfrac{13.8^2}{15}(13 + j.06) = 165.1 + j0.76$ ohms at 13.8 kV.

Case 2. Connections as above but load



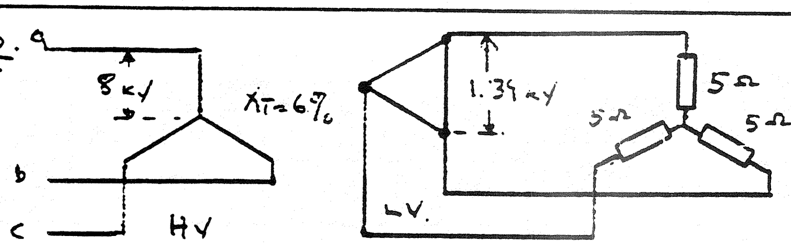
equivalent wye where $Z_L = \dfrac{5 \times 5}{15} = 1.67$ ohms per phase.

$Z_L = \dfrac{1.67}{.384} = 4.34$ pu at 15 MVA

$Z_{Total} = 4.34 + j.06$ pu at 15 MVA, 13.8 kV

$= \dfrac{13.8^2}{15}(4.34 + j.06) = 55.1 + j.76$ ohms at 13.8 kV.

Case 3.



HV Base $= 8\sqrt{3} = 13.8$ kV $\qquad$ LV Base $= 1.39$ kV

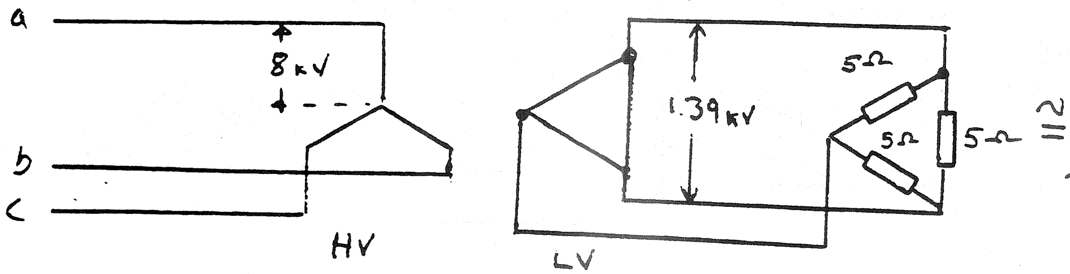
$Z_{B_H} = \dfrac{13.8^2}{15} = 12.7$ ohms at 13.8 kV. $\qquad Z_{B_L} = \dfrac{1.39^2}{15} = .129$ ohms at 1.39 kV

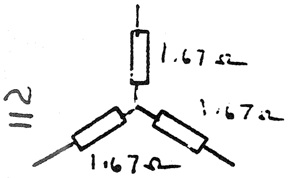
$Z_L = \dfrac{5}{.129} = 38.82$ pu. at 15 MVA

$Z_{Total} = 38.82 + j.06$ pu at 15 MVA 13.8 kV $= \dfrac{13.8^2}{15}(38.82 + j.06) = 493 + j.76$ ohms at 13.8 kV

Prob. 2.4 Cont'd.

Case 4:





$Z_{eq} = \frac{5 \times 5}{15} = 1.67\ \Omega$

$HV_{Base} = 8\sqrt{3} = 13\ kV$

$Z_{B_H} = \frac{13.8^2}{15} = 12.7$ ohms at 13.8 kV

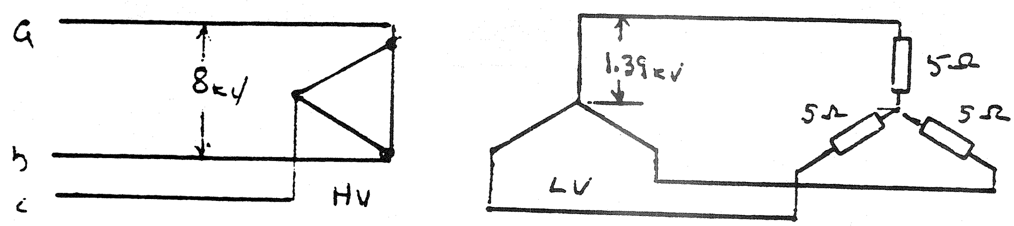
$LV_{Base} = 1.39\ kV$

$Z_{B_L} = \frac{1.39^2}{15} = 0.129$ ohms at 1.39 kV

$Z_L = \frac{1.67}{.129} = 13$ pu at 15 MVA

$Z_{Total} = 13 + j.06$ pu at 15 MVA 13.8 kV $= \frac{13.8^2}{15}(13 + j.06) = 165.1 + j.76$ ohms at 13.8 kV.

Case 5:



$HV_{Base} = 8\ kV$

$Z_{BH} = \frac{8^2}{15} = 4.27$ ohms at 8 kV

$LV_{Base} = 1.39\sqrt{3} = 2.4\ kV$

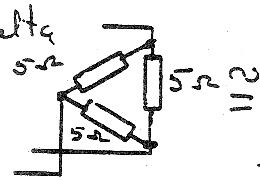
$Z_{B_L} = \frac{2.4^2}{15} = .384$ ohms at 2.4 kV

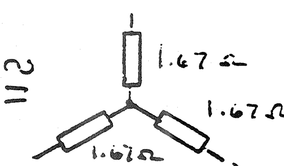
$Z_L = \frac{5}{.384} = 13$ pu at 15 MVA.

$Z_{Total} = 13 + j.06$ pu at 15 MVA 13.8 kV $= \frac{8^2}{15}(13 + j.06) = 55.2 + j.256$ ohms at 8 kV.

Case 6:

Same as 5 except load in delta





$Z_{eq} = \frac{5 \times 5}{15} = 1.67\ \Omega$

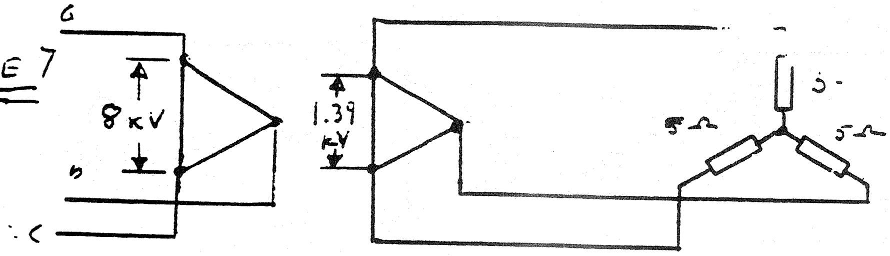
$Z_L = \frac{1.67}{.384} = 4.34$ pu at 15 MVA

$Z_{Total} = 4.34 + j.06$ pu at 15 MVA

$= \frac{8^2}{15}(4.34 + j.06) = 18.6 + j.256$ ohms at 8 kV.

Problem 2.4 Cont'd.

CASE 7



$Z = \frac{\;}{.129}$

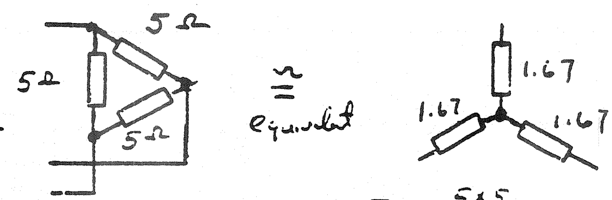
$HV_{Base} = 8\text{ kV}$ $LV_{Base} = 1.39\text{ kV}$

$Z_{BH} = \frac{8^2}{15} = 4.27$ ohms at 8 kV $Z_{BL} = \frac{1.39^2}{15} = 0.129$ ohms at 1.39 kV

$Z_{TOTAL} = 38.82 + j.06$ pu at 15 kV $= \frac{8^2}{15}(38.82 + j.06) = 165.6 + j.256$ ohms at 8 kV

Case 8

Connections as Case 7 above but load in delta



$Z_L = \frac{5 \times 5}{15} = 1.67$ ohms.

$HV_{Base} = 8\text{ kV}$ $LV_{Base} = 1.39\text{ kV}$

$Z_{BH} = \frac{8^2}{15} = 4.27$ ohms at 8 kV $Z_{BL} = \frac{1.39^2}{15} = 0.129$ ohms at 1.39 kV.

$Z_L = \frac{1.67}{.129} = 13$ pu at 15

$Z_{TOTAL} = 13 + j.06$ pu at 15 MVA $= \frac{8^2}{15}(13 + j.06) = 55.2 + j.256$ ohms at 8 kV.

Problem 2.5 page 501. Reactances on 100 MVA 13.8 kV or 115 kV.

Generator: $X_d'' = \frac{100}{50}\left(\frac{13.2}{13.8}\right)^2 \times .20 = 0.366$ pu.

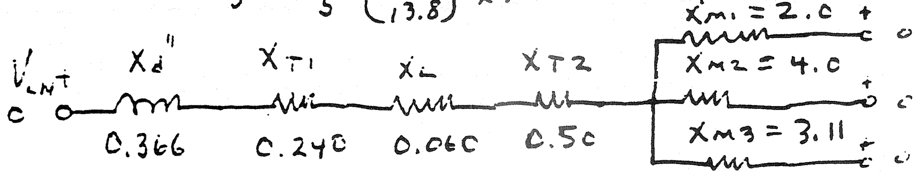
Transf. #1: $X_{T1} = \frac{100}{50} \times .12 = 0.240$ pu; Transf. #2 $X_{T2} = \frac{100}{20} \times .10 = 0.50$ pu

Line: $X_L = \frac{MVA\, Z_\Omega}{kV^2} = \frac{100 \times 0.5 \times 16}{115^2} = 0.060$ pu.

Motor M1; $X_{M1} = \frac{100}{10} \times .20 = 2.0$ pu

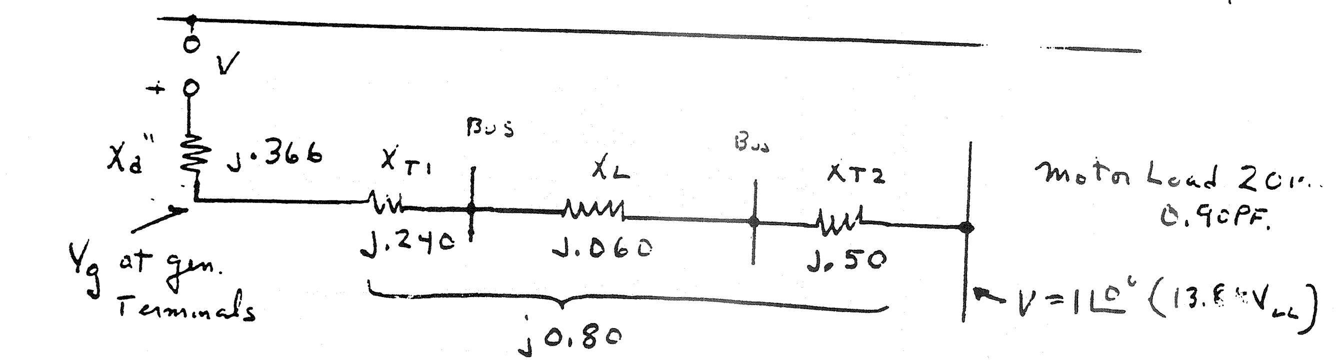
Motor M2; $X_{M2} = \frac{100}{5} \times .20 = 4.0$ pu.

Motor M3; $X_{M3} = \frac{100}{5}\left(\frac{13.2}{13.8}\right)^2 \times .17 = 3.11$ pu.

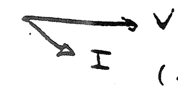


Reactances in per unit at 100 MVA 13.8 kV or 115 kV.

Prob. 2.6 page 501.   From Prob. 2.[illegible], ([illegible] on 100 MVA. in per unit.) Pg 5



$$I_{load} = \frac{20}{100} = 0.20 \angle -25.84^\circ \text{ pu}$$



(.9PF = 25.84

$$V_g = 1.0\angle 0^\circ + I_{load}(j.80) = 1.0\angle 0^\circ + .20 \times .80 \angle 90^\circ - 25.84^\circ$$

$$= 1.0 + .16 \angle 64.16^\circ = 1.0 + .0697 + j.144$$

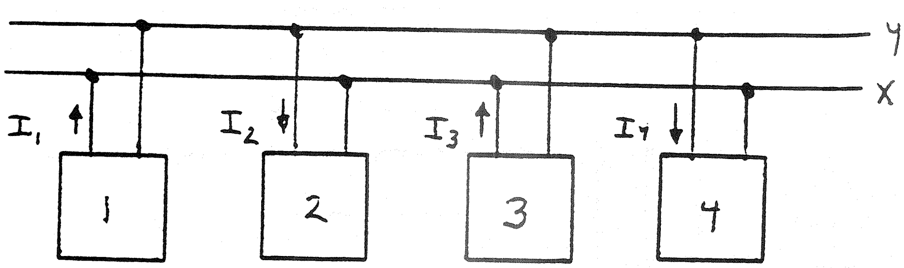
$$= 1.0697 + j.144 = 1.079 \angle 7.67^\circ \text{ pu}$$ or 14.896 kV line to line.

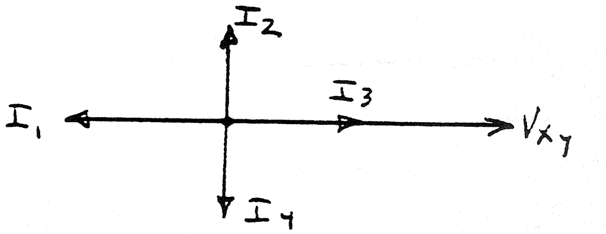
$$V = 1.0 + .20(.366 + .80) \angle 90^\circ - 25.84^\circ = 1.0 + .233 \angle 64.16^\circ$$

$$= 1.0 + .1017 + j.2099 = 1.1017 + j.2099 = 1.12 \angle 10.786^\circ \text{ pu}$$

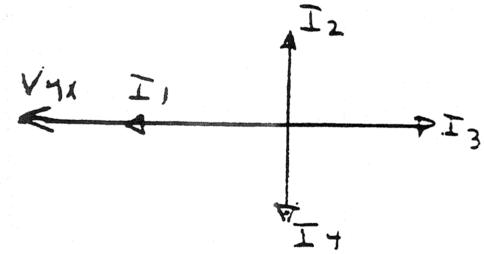
= 15.476 kV line to line inside machine

Problem 3.1 page 502





1. Resistor
2. Reactor
3. Generator
4. Capacitor



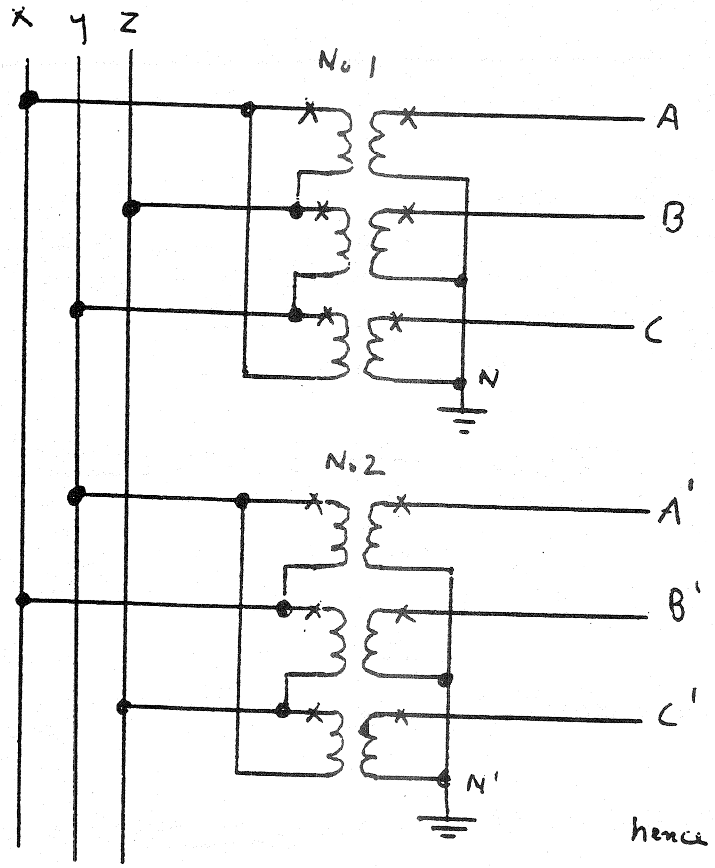
I_1 from y to x is in phase with V_{yx} = Resistor

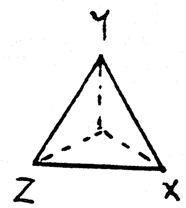
I_2 " y to x lags V_{yx} = reactor

I_3 " y to x is 180° from V_{yx} = generator

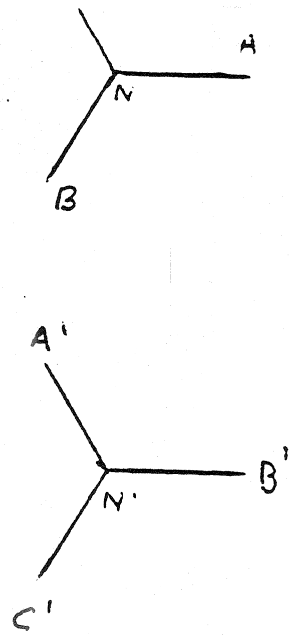
I_4 " y to x leads V_{yx} = Capacitor.

Problem 3.2 pages 502-503.



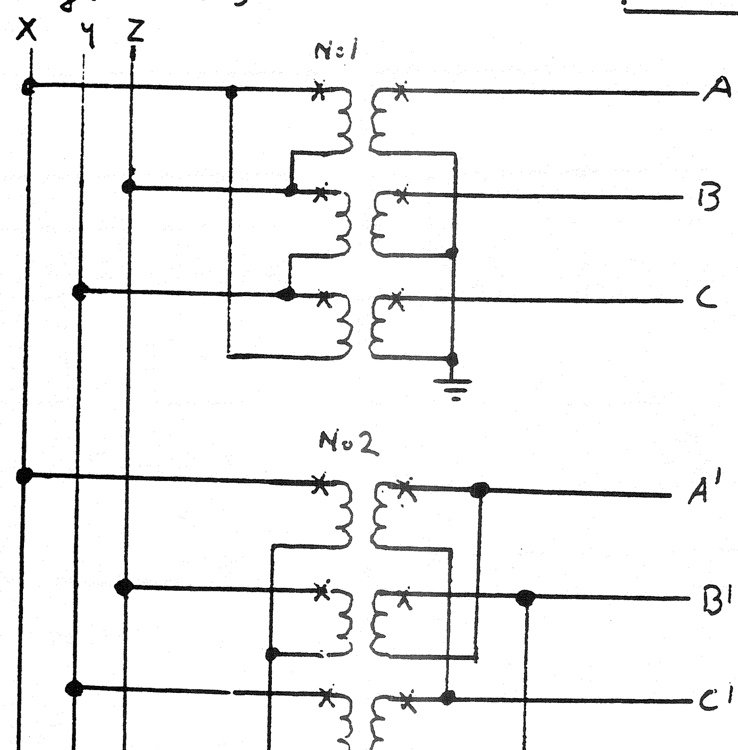


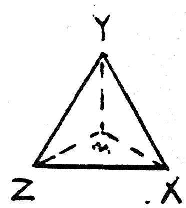
assume

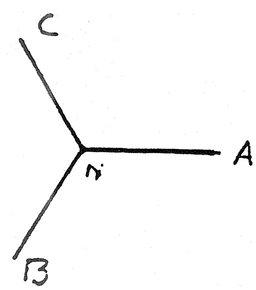


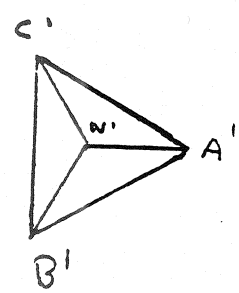
hence $V_{A'N'}$ leads V_{AN} 120°
$V_{B'N'}$ leads V_{BN} 120°
$V_{C'N'}$ leads V_{CN} (

Problem 3.3 page 502-503



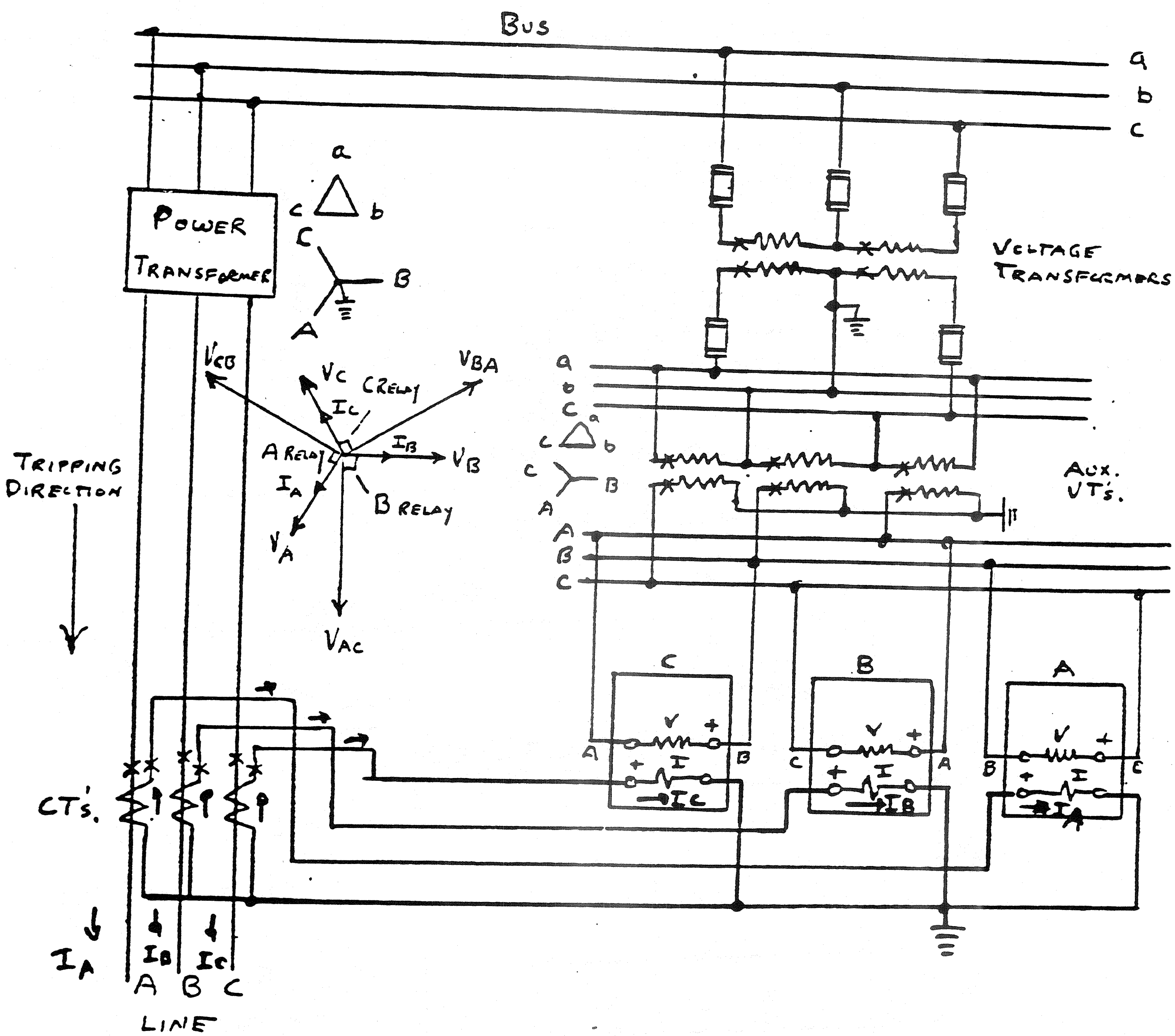






other connections are possible.

Problem 3.4 page 504



Problem 4.1 pg 505.

analysis by examination	
Right Side Bus H Currents.	Since all phase currents are equal & in phase: zero Sequence ONLY
Left Side Bus G Currents	With no positive seq. on right side, Pos. Seq must exist on left. No Zero Sequence since currents add to zero. With unbalanced Currents neg. Seq. must exist.

analysis by basic equations 4.8, 4.9 & 4.10

$I_0 = \frac{1}{3}(1+1+1) = 1.0$ pu

$I_1 = \frac{1}{3}(1+a+a^2) = 0$

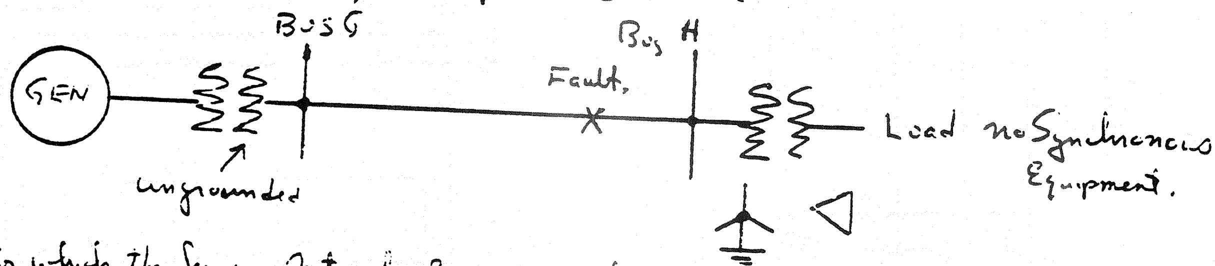
$I_2 = \frac{1}{3}(1+a^2+a) = 0$

$I_0 = \frac{1}{3}(2-1-1) = 0$

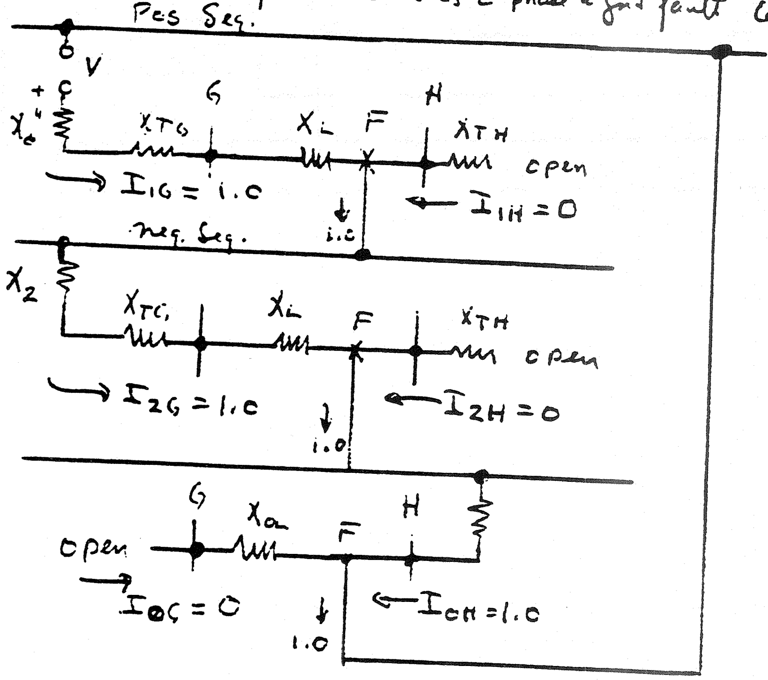
$I_1 = \frac{1}{3}(2-a-a^2) = 1.0$ pu

$I_2 = \frac{1}{3}(2-a^2-a) = 1.0$ pu.

From above analysis, The System could be:



For which The Sequence Networks & phase to gnd fault connections are



$I_1 = I_2 = I_0 = \frac{V}{X_1 + X_2 + X_0} = 1.0$ pu.

$I_{aF} = 3I_0 = 3.0$ pu.

Check! Bus G Side — equations 4.5, 4.6 and 4.7

$I_{aG} = 1.0 + 1.0 + 0 = 2.0$

$I_{bG} = a^2 + a + 0 = -1.0$

$I_{cG} = a + a^2 + 0 = -1.0$

as shown in problem.

Bus H side

$I_{aH} = 0 + 0 + 1.0 = 1.0$

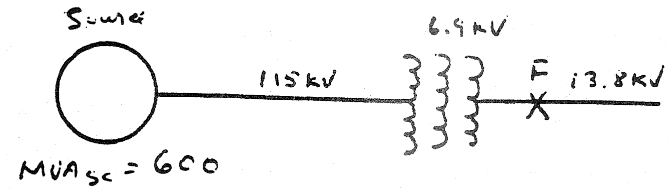
$I_{bH} = 0 + 0 + 1.0 = 1.0$

$I_{cH} = 0 + 0 + 1.0 = 1.0$

as shown in Problem.

Prob. 4.2 pg 505-506. Per unit on 30MVA. (Pg

a)



Y Δ Y

eq. A4.1-4 pg 138.

$X_{source} = \frac{30}{600} = 0.05$ pu

$X_{115-13.8} = 10\% = 0.10$ pu.

$X_{115-6.9} = \frac{30}{15} \times 22.5 = 45\% = .45$

$X_{13.8-6.9} = \frac{30}{10} \times 11 = 33\% = .33$ pu.

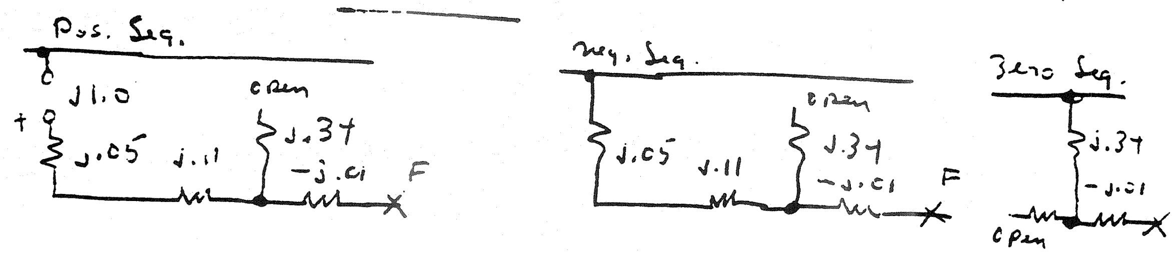
From eqs. A4-2-13, -14 & -15 pg 146.

$X_{115} = \frac{1}{2}(.10 + .45 - .33) = 0.11$ pu

$X_{13.8} = \frac{1}{2}(.10 + .33 - .45) = -0.01$ pu

$X_{6.9} = \frac{1}{2}(.45 + .33 - .10) = 0.34$ pu.

b)



$X_1 = X_2 = j\,0.15$ pu.

$X_0 = j.33$ pu.

c) $I_{3\phi} = \frac{j1.0}{j.15} = 6.67 \text{ pu} = \frac{30{,}000}{\sqrt{3}\,13.8} 6.67 = 8367.4$ amp, at 13.8 kV.

d) $I_1 = I_2 = I_0 = \frac{j.10}{j(.15 + .15 + .33)} = \frac{1.0}{0.63} = 1.587$ pu.

$I_a = 3I_0 = 4.762 \text{ pu} = \frac{30{,}000}{\sqrt{3}\,13.8} \times 4.76 = 5976.7$ amps at 13.8 kV.

e) Voltages on 13.8 kV side: for fault of part d:

$V_1 = j1.0 - j.15 \times 1.587 = j.7619$ pu

$V_2 = 0 - j.15 \times 1.587 = -j.2381$ pu

$V_0 = 0 - j.33 \times 1.587 = -j.5238$ pu.

$V_{aF} = V_1 + V_2 + V_0 = 0$

$V_{bF} = a^2V_1 + aV_2 + V_0 = .7619\angle 90+240 + .2381\angle -90+120 - j.5238$

$= .8660 - j.7857 = 1.169\angle 42.22^\circ$ pu $= \frac{13{,}800}{\sqrt{3}} 1.169 = 9313.9$ volts LN

$V_{cF} = aV_1 + a^2V_2 + V_0 = .7619\angle 90+120 + .2381\angle -90+240 - j.5238$

$= -.8660 - j.7857 = 1.169\angle -137.75^\circ$ pu $= 9313.9$ volts LN.

normal $V_{LN} = \frac{13{,}800}{\sqrt{3}} = 7967.43$ volts.

Prob. 4.2 cont'd.

f) Values on the 115 kV side: no phase shift; $N = \frac{115}{13.8} = 8.33$

For the fault of Part d. $I_1 = I_2 = 1.587$ pu. $I_0 = 0$

$I_{a115} = I_1 + I_2 + I_0 = 3.174$ pu $= \frac{30{,}000}{\sqrt{3} \times 115} \times 3.17 = 478.14$ amps at 115 kV

$I_{b115} = a^2 I_1 + a I_2 + I_0 = -I_1 = -1.587$ pu $= -239.07$ amps at 115 kV

$I_{c115} = a I_1 + a I_2 + I_0 = -I_1 = -1.587$ pu $= -239.07$ amps at 115 kV

Correctly, $3I_0 = 0 = I_a + I_b + I_c$

$V_{1\,115} = j1.0 - j.05 \times 1.587 = j.9206$ pu.

$V_{2\,115} = 0 - j.05 \times 1.587 = -j.0794$ pu.

$V_{0\,115} = 0$

$V_{a115} = V_1 + V_2 + V_0 = j.8412$ pu $= \frac{115{,}000}{\sqrt{3}} \times .8412 = 55{,}851.71$ volts LN.

$V_{b115} = a^2 V_1 + a V_2 + V_0 = .9206 \angle 90+240 + .0794 \angle -90+120$

$= .8661 - j.4206 = .9628 \angle -25.9^\circ$ pu $=$ 63,927.09 volts LN.

$V_{c115} = a V_1 + a^2 V_2 + V_0 = .9206 \angle 90+120 + .0794 \angle -90+240$

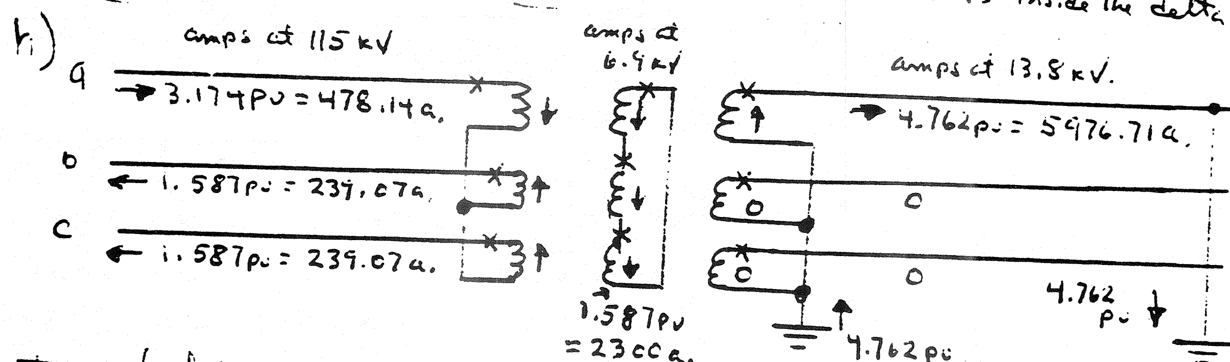
$= -.8661 - j.4206 = .9628 \angle -154.1^\circ$ pu $=$ 63,927.09 volts LN.

g) Current in delta Tertiary for fault of Part d.

$I_0 = 1.587$ pu $= 1.587 \frac{30{,}000}{3 \times 6.9} = 2300$ amp at 6.9 kV

Note 3 not $\sqrt{3}$ inside the delta

h)



Ampere Turn check: 115 kV winding 6.9 kV winding 13.8 kV winding

Assume Per unit Turns = 1.0 Then $\frac{6.9\sqrt{3}}{115} = .1039$ Turn & $\frac{13.8}{115} = .120$ Turn

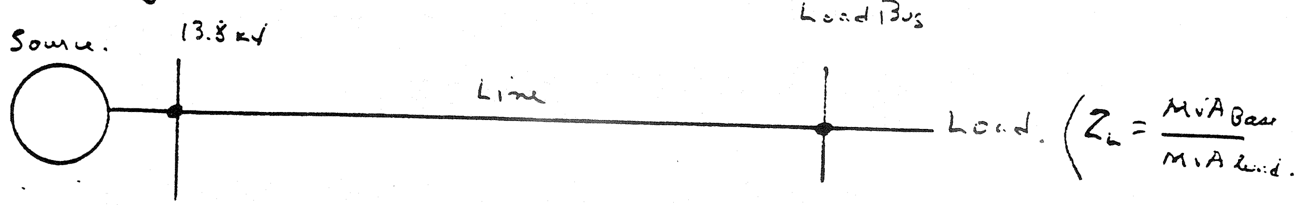
Phase a check! Currents flowing down vs Current flowing up.

478.14 × 1 = 478.14 AT

2300 × .1039 = 239.07 AT

Total = 717.21 AT. ← check. → 5976.71 × .12 = 717.21.

Prob. 4.3 pg 506   Per unit on 100MVA.



$X_1 = X_2 = \frac{100}{312.5} = j.32$ pu. $X_1 = X_2 = j.42$ pu $Z_{load} = \frac{100}{4} = 25\angle 25.8419°$

$X_0 = j1.26$ pu.

$Z_0 = 11.93\angle 8°$ pu $= 22.50 + j10.8973$ pu

$= 11.8139 + j1.66034$ pu.

a) Normal load currents!   V of source $= 1.0\angle 0°$ pu.

Total $Z = j.32 + j.42 + 22.5 + j10.897 = 22.5 + j11.64 = 25.33\angle 27.35$

$I_{load} = \frac{1}{25.33} = .0395\angle -27.35°$ pu. $= \frac{100{,}000}{\sqrt{3}\ 13.8} \times .0395 = 165.26$ amps at 13.8 kV

b) V_L at load bus!

$V_L = 1.0 - .0395(.32 + .42)\angle -27.35 + 90°$

$= .987 - j.026 = .987\angle -1.51°$ pu.

c) Phase a-to-ground fault, neglecting load, at Load bus.

$X_1 = X_2 = j.32 + j.42 = j.74$ pu $X_0 = 11.814 + j2.92$ pu

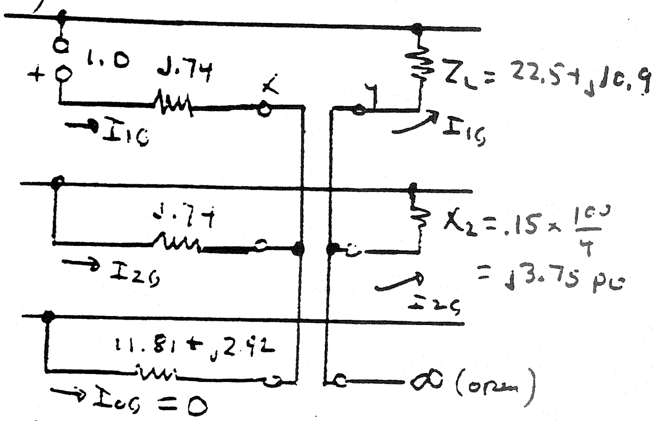
$I_{1G} = I_{2G} = I_{0G} = \frac{1\angle 0°}{j.74 + j.74 + 11.814 + j2.92} = \frac{1}{11.81 + j4.4} = \frac{1}{12.60\angle 20.430}$ pu

$= .079\angle -20.43°$ pu

$I_{aG} = 3I_{0G} = .237\angle -20.43°$ pu $I_{bG} = I_{cG} = 0$ pu

Ground Fault is 6 times load current magnitude.

d) Phase a open at the load Bus.



$I_{1G} + I_{2G} = 0$

or $I_{2G} = -I_{1G}$.

Prob. 4.3 d Cont'd. : voltage drop around the circuit:

$$-1.0 + j.74\, I_{1G} - j.74\, I_{2G} - j3.75\, I_{2G} + (22.5 + j10.9)\, I_{1G} = 0$$

Combining & substituting $-I_{2G} = I_{1G}$;

$$I_{1G} = \frac{1.0}{27.68 \angle 35.64} = .036 \angle -35.64^\circ \text{ pu.}$$

$$I_{2G} = .036 \angle 144.36^\circ \text{ pu.}$$

$I_{aG} = I_{1G} + I_{2G} = 0$ which it should be with a phase open.

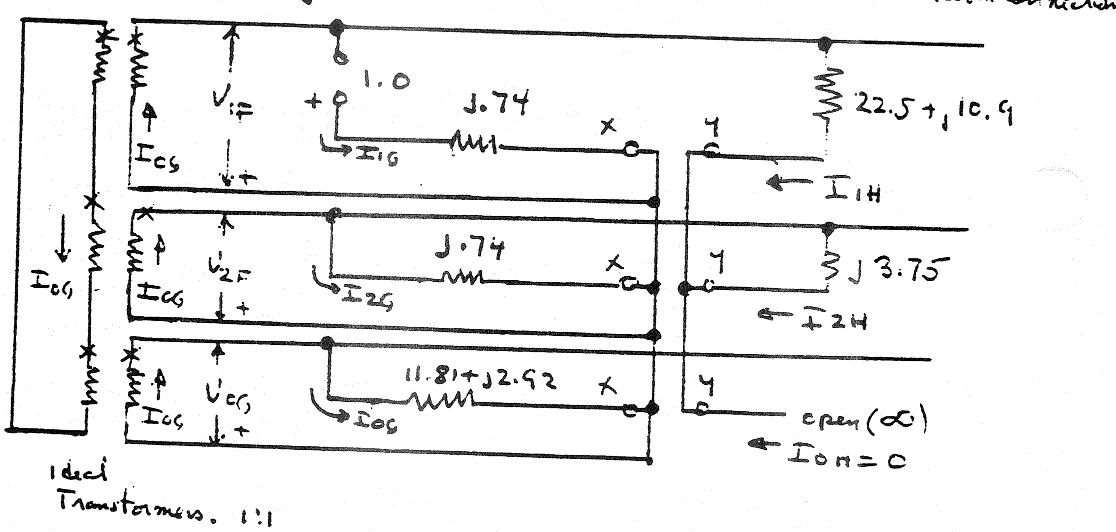
$I_{bG} = a^2 I_{1G} + a I_{2G} = (a^2 - a) I_{1G} = -j\sqrt{3}\, I_{1G} = .062 \angle -125.64^\circ$ pu

$I_{CG} = a I_{1G} + a^2 I_{2G} = (a - a^2) I_{1G} = j\sqrt{3}\, I_{1G} = .062 \angle 54.36^\circ$ pu.

The two phase currents are 1.57 times load current magnitude.

---

e) Open phase a and grounded on the Source side. Network Connections are



$I_{1G} + I_{1H} = I_{OG} = I_{2G} + I_{2H}$ OR $I_{1G} - I_{2G} = I_{2H} - I_{1H}$ (1)

$I_{1H} + I_{2H} = 0$ OR $I_{2H} = -I_{1H}$ (2)

Thus (1) becomes $I_{1G} - I_{2G} = -2 I_{1H}$ (3)

Voltage Drops around pos & neg. sequence networks:

$$-1.0 + j.74 I_{1G} - j.74 I_{2G} + j3.75 I_{2H} - (22.5 + j10.9) I_{1H} = 0 \quad (4)$$

Substituting (2) & (3) in (4)

$$-1.0 + j.74(-2I_{1H}) + j3.75(-I_{1H}) - (22.5 + j10.9) I_{1H} = 0$$

$$-j1.48 I_{1H} - j3.75 I_{1H} - (22.5 + j10.9) I_{1H} = 1.0 \text{ OR } (-22.5 - j16.13) I_{1H} = 1.0$$

$$I_{1H} = \frac{1.0}{27.68 \angle -144.36} = .036 \angle 144.36^\circ \text{ pu} \;\; \& \;\; I_{2H} = .036 \angle -35.64^\circ \text{ pu.} \quad (5)$$

$$= -.029 + j.021 \text{ pu} \qquad = .029 - j.021 \text{ pu}$$

Voltage Drops from network: Pos Seq: $-1.0 + j.74 I_{1G} + V_{1F} = 0$

neg Seq. $0 + j.74 I_{2G} + V_{2F} = 0$

$11.81 + j2.92 I_{0G} + V_{0F} = 0$

and $V_{1F} + V_{2F} + V_{0F} = 0$ So adding above:

$$-1.0 + j.74(I_{1G} + I_{2G}) + (11.81 + j2.92) I_{0G} = 0 \qquad (6)$$

also $I_{1G} + I_{2G} + I_{0G} = 3I_{0G}$ OR $I_{1G} + I_{2G} = 2I_{0G}$ (7)

Substituting (7) in (6) and solving for I_{0G}:

$$j.74(2I_{0G}) + 11.81 + j2.92 I_{0G} = 1.0$$

$$I_{0G}(11.81 + j4.40) = 1.0 \qquad I_{0G} = \frac{1}{12.6\angle 20.43°} = .079\angle{-20.43°} \text{ pu}$$

$$= .074 - j.028 \text{ pu} \qquad (8)$$

$$I_{1G} = I_{0G} - I_{1H} = .074 - j.028 + .029 - j.021 = .103 - j.049 = .114\angle{-25.44°} \text{ pu}$$

$$I_{2G} = I_{0G} - I_{2H} = .074 - j.028 - .029 + j.021 = .045 - j.007 = .046\angle{-8.84°} \text{ pu}$$

$$I_{aG} = .103 - j.049 + .045 - j.007 + .074 - j.028 = .222 - j.084 = .237\angle{-20.73°} \text{ pu}$$

$$I_{bG} = .114\angle{-25.44+240} + .046\angle{-8.84+120} + .074 - j.028$$

$$= -.097 - j.065 - .017 + j.043 + .074 - j.028 = -.037 - j.05 = .062\angle{-126.50°} \text{ pu}$$

$$I_{cG} = .114\angle{-25.4+120} + .046\angle{-8.84+240} + .074 - j.028$$

$$= -.009 - j.114 - .029 - j.036 + .074 - j.028 = +.037 + j.05 = .062\angle 53.50° \text{ pu.}$$

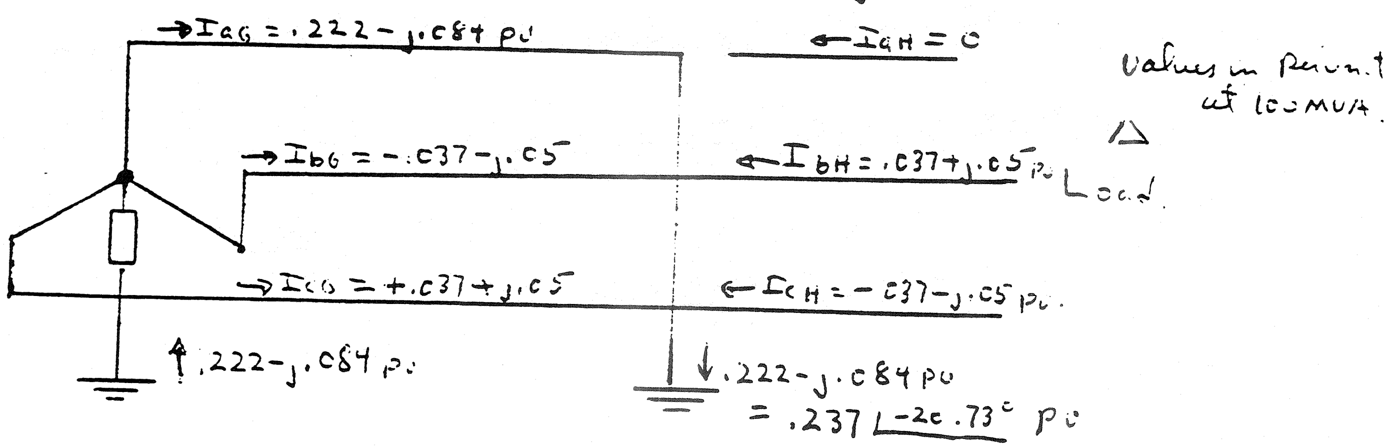
$I_{aH} = 0$ as it should be.

$$I_{bH} = .036\angle{144.36+240} + .036\angle{-35.64+120}$$

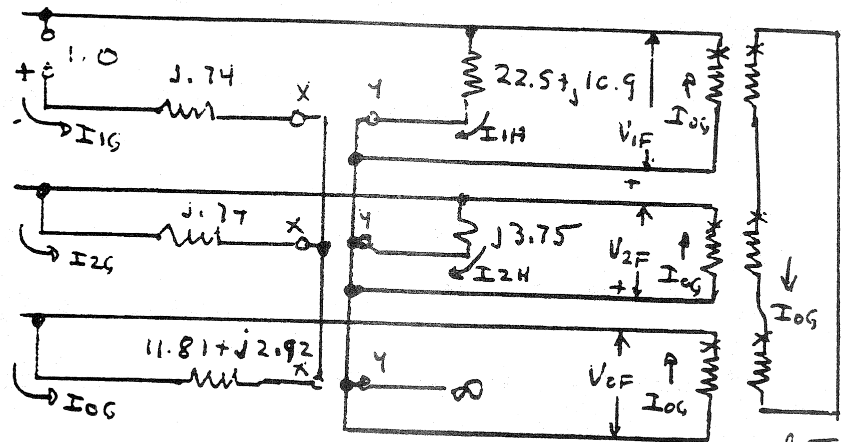
$$= .033 + j.015 + .004 + j.036 = .037 + j.05 = .062\angle 53.50° \text{ pu}$$

$$I_{cH} = .036\angle{144.36+120} + .036\angle{-35.64+240}$$

$$= -.004 - j.036 - .033 - j.015 = -.037 - j.05 = .062\angle{-126.50°} \text{ pu.}$$



Prob. 4.3 f Open phase a and grounded on Load side



Ideal Transformer 1:1

$I_{1G} + I_{2G} + I_{0G} = 0$ & $V_{1F} + V_{2F} + V_{0F} = 0$ (1)

$I_{1G} + I_{1H} = I_{0G}$ & $I_{2G} + I_{2H} = I_{0G}$ (2)

So $I_{2H} - I_{1H} = I_{1G} - I_{2G}$ (3)

Drops around Pos. & Neg. Seq. networks:

$-1.0 + j.74 I_{1G} - j.74 I_{2G} + j3.75 I_{2H} - (22.5 + j10.9) I_{1H} = 0$ (4)

Substituting (3) in (4):

$-1.0 + j.74 I_{2H} - j.74 I_{1H} + j3.75 I_{2H} - (22.5 + j10.9) I_{1H} = 0$

$-(22.5 + j11.64) I_{1H} + j4.49 I_{2H} = 1.0$ (5)

Drops around neg. & zero seq. networks:

$j.74 I_{2G} - (11.81 + j2.92) I_{0G} - V_{0F} - j3.75 I_{2H} = 0$ (6)

now $V_{0F} = -V_{1F} - V_{2F}$ where $-V_{1F} = (22.5 + j10.9) I_{1H}$; $-V_{2F} = j3.75 I_{2H}$

So $V_{0F} = (22.5 + j10.9) I_{1H} + j3.75 I_{2H}$ (7)

also: $3 I_{0G} = I_{1H} + I_{2H}$ or $I_{0G} = \frac{1}{3} I_{1H} + \frac{1}{3} I_{2H}$ (8)

From (2): $I_{2G} = I_{0G} - I_{2H} = \frac{1}{3} I_{1H} - \frac{2}{3} I_{2H}$ (9)

Substituting (7), (8) & (9) in (6):

$j.74\left(\frac{1}{3} I_{1H} - \frac{2}{3} I_{2H}\right) - (11.81 + j2.92)\left(\frac{1}{3} I_{1H} + \frac{1}{3} I_{2H}\right) - (22.5 + j10.9) I_{1H} - j3.75 I_{2H} - j3.75 I_{2}$

$j.24667 I_{1H} - j.49333 I_{2H} - (3.93667 + j.97333) I_{1H} - (3.93667 + j.97333) I_{2H}$

$-(22.5 + j10.9) I_{1H} - j7.5 I_{2H} =$

$(-26.43667 - j11.62666) I_{1H} - (3.93667 + j8.96666) I_{2H} = 0$ (10)

$28.88039 \angle{-156.26045^\circ}\ I_{1H} = 9.79277 \angle{66.29688^\circ}\ I_{2H}$

$I_{2H} = \frac{28.88039}{9.79277} \angle{-156.26045^\circ - 66.29688}\ I_{1H} = 2.94915 \angle{-222.55733^\circ}\ I_{1H}$ (1

Prob 4.3f Cont'd

Substitute (11) in (2)

$-(22.5+j11.64)I_{1H} + 4.49 \times 2.94915 \underline{/90-222.55733^\circ}\ I_{1H} = 1.0$ (13.24168; -132.55733°)

$(-22.5-j11.64-8.95572-j9.75384)I_{1H} = 1.0 = (-31.45572-j21.39384)I_{1H}$

$I_{1H} = \dfrac{1.0}{38.04154\angle} = .02629\underline{/145.77938^\circ}$ pu $= -.02174+j.01478$ (12)

$I_{2H} = 2.94915 \times .02629 \underline{/-222.55733+145.77938}$

$= .07753\underline{/-76.77795}$ pu $= +.01773-j.07548$ (13)

$I_{0G} = \frac{1}{3}(I_{1H}+I_{2H}) = -.00134-j.02023 = .02028\underline{/-93.77962^\circ}$

$3I_{0G} = -.00401-j.06070 = .06083\underline{/-93.77962^\circ}$ pu (14)

$I_{1G} = I_{0G}-I_{1H} = -.00134-j.02023+.02174-j.01478$

$= .02040-j.03501 = .04052\underline{/-59.77104^\circ}$ pu (15)

$I_{2G} = I_{0G}-I_{2H} = -.00134-j.02023-.01773+j.07548$

$= -.01907+j.05525 = .05845\underline{/109.042540^\circ}$ pu (16)

check $I_{1G}+I_{2G}+I_{0G} = 0 = .02040-j.03501-.01907+j.05525-.00134-j.02023$ ok

$I_{aG} = I_{1G}+I_{2G}+I_{0G} = 0$ as it should be.

$I_{bG} = a^2I_{1G}+aI_{2G}+I_{0G} = .04052\underline{/-59.77104+240}$ (180.22896) $+ .05845\underline{/109.04254+120}$ (229.04254) $+ I_{0G}$

$= -.04052-j.00016-.03831-j.04414-.00134-j.02023$

$= -.08017-j.06453 = .10291\underline{/141.16882^\circ}$ pu.

$I_{cG} = aI_{1G}+a^2I_{2G}+I_{0G} = .04053\underline{/-59.77104+120}$ (60.22896) $+ .05845\underline{/109.04254+240}$ (349.04254) $+ I_{0G}$

$= +.02012+j.03518+.05738-j.01111-.00134-j.02023$

$= .07616+j.00384 = .07626\underline{/2.88642^\circ}$ pu

$3I_0 = I_{aG}+I_{bG}+I_{cG} = -.08017-j.06453+.07616+j.00384 = -.00401-j.06069$ ch

$I_{aH} = I_{1H}+I_{2H}+I_{0H} = -.02174+j.01478+.01773-j.07548 = -.00401-j.06070 = 3I$ (1

$I_{bH} = a^2I_{1H}+aI_{2H}+0 = .02629\underline{/145.77938+240}$ (25.77938) $+ .07753\underline{/-76.77795+120}$ (43.22205)

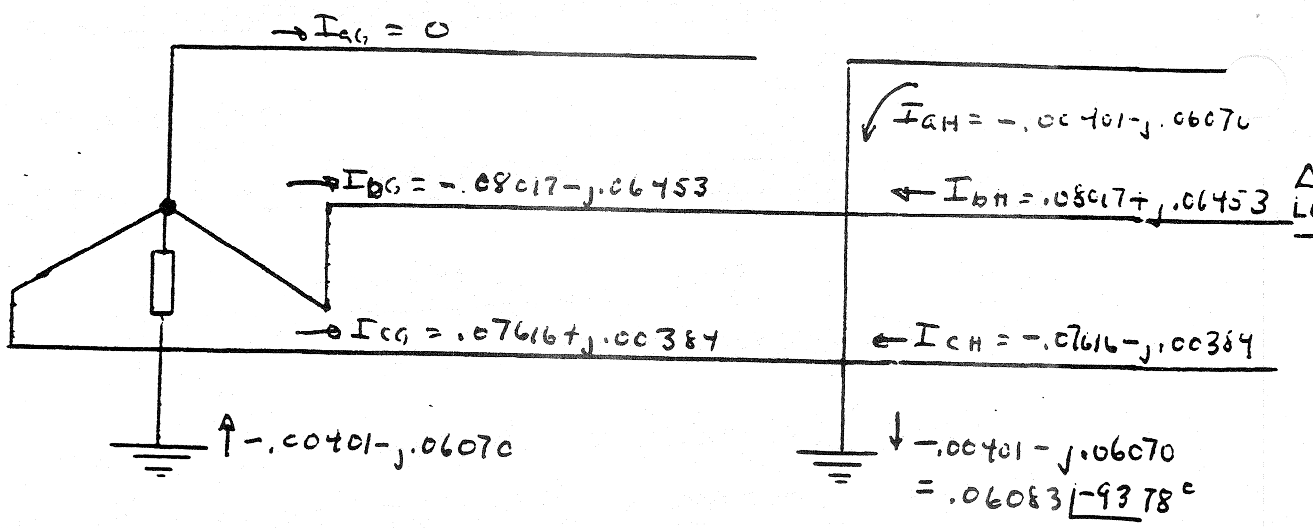
$= .02367+j.01143+.05650+j.05309 = .08017+j.06453 = I_{bG}\underline{/180^\circ}$

$I_{cH} = aI_{1H}+a^2I_{2H}+0 = .02629\underline{/145.77938+120}$ (265.77938) $+ .07753\underline{/-76.77795+240}$ (169.22205)

$= -.00193-j.02622-.07423+j.02238 = -.07616-j.00384 = I_{cG}\underline{/180^\circ}$

Prob. 4.3f Cont'd.

Values in per unit at 100 MVA



Summary for Prob. 4.3 (a) $I_{load} = .0395 \angle -27.35°$ pu

3φ Fault. $I_{3\phi F} = \frac{1}{.74} = 1.35 \angle 90°$ pu ($34.2 \times I_{load}$)

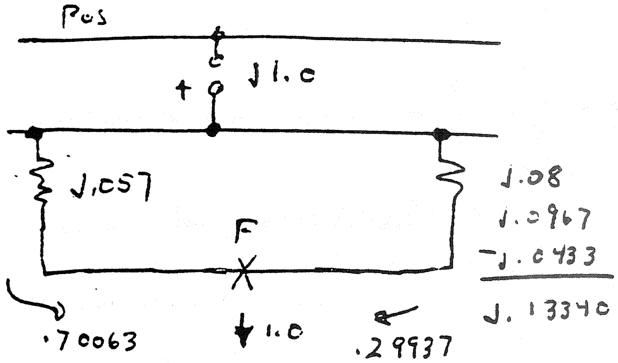
(c) $I_{\phi GF} = .237 \angle -20.43°$ pu ($6 \times I_{load}$)

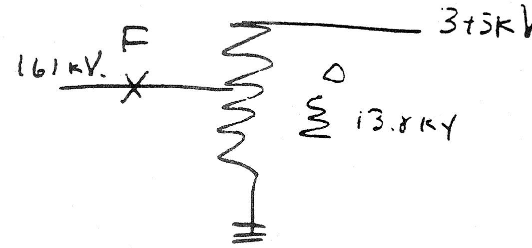
open phase (d) $I_{bG} = .062 \angle -125.64°$; $I_{cG} = .062 \angle 54.36°$ ($1.57 \times I$

Open phase, gnd on Same (e) $I_{aF} = .237 \angle -20.73°$ pu ($6 \times I_{load}$)

open Phase, gnd on Load Side (f) $I_{aF} = .06083 \angle -93.78°$ pu ($1.54 \times I_{load}$)

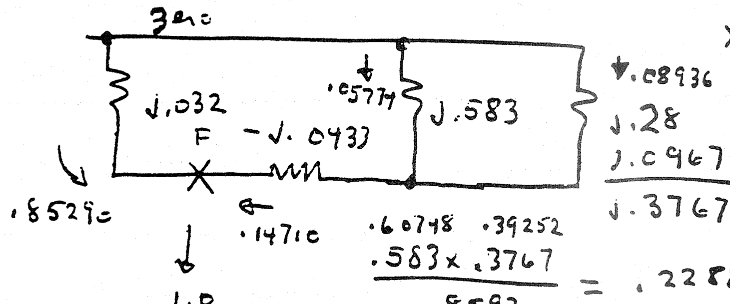
Prob. 4.4 pg 507. From Section 4.14 fig 4.26 pg 117.
on 100 MVA.





$$X_1 = \frac{.057 \times .1334}{.19040} = j.03994 \text{ pu}$$

$X_2 = j.03994$ pu.



$$\frac{.583 \times .3767}{.95970} = .22884$$

$$.22884 - .0433 = .18554$$

$$X_0 = \frac{.032 \times .18554}{.21754} = j.0272$$

$$I_1 = I_2 = I_0 = \frac{j1.0}{j(.03994 + j.03994 + j.02729)} = \frac{1.0}{.10717} = 9.33097 \text{ pu}$$

$$= \frac{100{,}000}{\sqrt{3} \times 161} 9.33097 = 3346.11058 \text{ amps at } 161 \text{ kV}.$$

$$I_{aF} = 3I_0 = 27.99291 \text{ pu} = 10{,}038.33175 \text{ amp. at } 161 \text{ kV}.$$

amps at 161 kV

$I_1 = I_2 = 3346.11 \times .70063 = 2344.38546$ a.

$I_0 = 3346.11 \times .85290 = 2853.89772$ a.

$I_1 = I_2 = 3346.11 \times .29937 = 1001.72513$ a.

$I_0 = 3346.11 \times .14710 = 492.21287$

amps at 345 kV

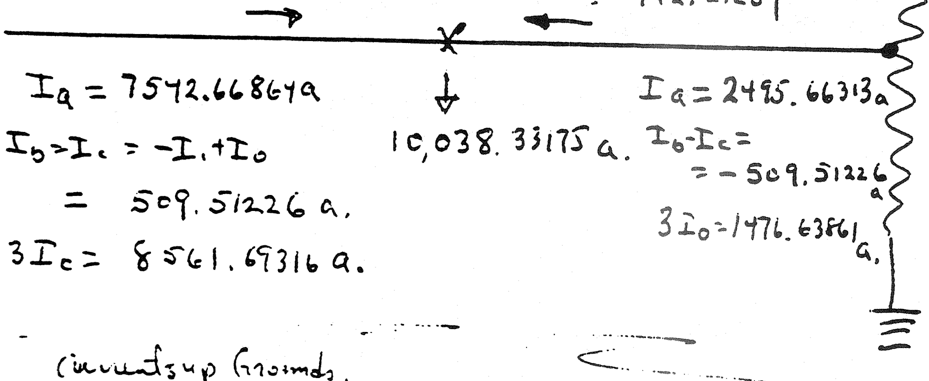
$I_1 = I_2 = \frac{161}{345} \times 3346.11 \times .29937 = 467.4$

$I_0 = \frac{161}{345} \times 3346.11 \times .08936 = 139.53$

$I_a = 1074.48073$ a

$I_b = I_c = -I_1 + I_0 = -327.93446$ a.

$3I_0 = 418.61181$ a.



$I_a = 7542.66864$ a

$I_b = I_c = -I_1 + I_0 = 509.51226$ a.

$3I_0 = 8561.69316$ a.

10,038.33175 a.

$I_a = 2495.66313$ a

$I_b = I_c = -509.51226$ a

$3I_0 = 1476.63861$ a.

(next pg)

$3(492.2187 - 139.53727) = 1058.02680$ amperes.

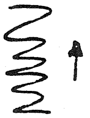
Currents up Grounds.

8561.69316
1058.02680
418.61181
10,038.33177 = Fault current into Ground.

Prob. 4.4 Cont'd

Current in Tertiary

13.8 kV.



$$I_0 = \frac{345}{\sqrt{3}\,13.8} \times 3346.11053 \times .05774$$

$= 2788.66567$ amps at 13.8 kV. in correct direction

For 345 kV faults: current down auto transformer neutral & reversed in Tertiary

For 161 kV faults: current up auto transformer neutral & correct in Tertiary.

Neither current can be used for polarizing ground relays for this Case.

Prob 5.1 pg 507. Fig. 5-10 on pg 167. (Background was omitted So Curve enclosed.)

(A) 100:5 = 20:1 Load of 80 amps. assume $\frac{80}{20} = 4$ amps sec.

$1.25 \times I_{Load} = 1.25 \times 4 = 5$ amp. - why Tap 5 chosen:

at Tap 5 Relay Burden $= \frac{3.2}{5^2} = 0.128$ ohm.

from Table CT Sec. Burden = 0.082 "

Lead Burden = 0.380 "

Total Z_B = 0.590 ohm.

$V_S = I_S Z_B = 5 \times .590 = 2.95$ volts. I_e from fig 10 = 0.23 amps.

assuming direct addition $I_S' = I_S + I_e = 5.23$

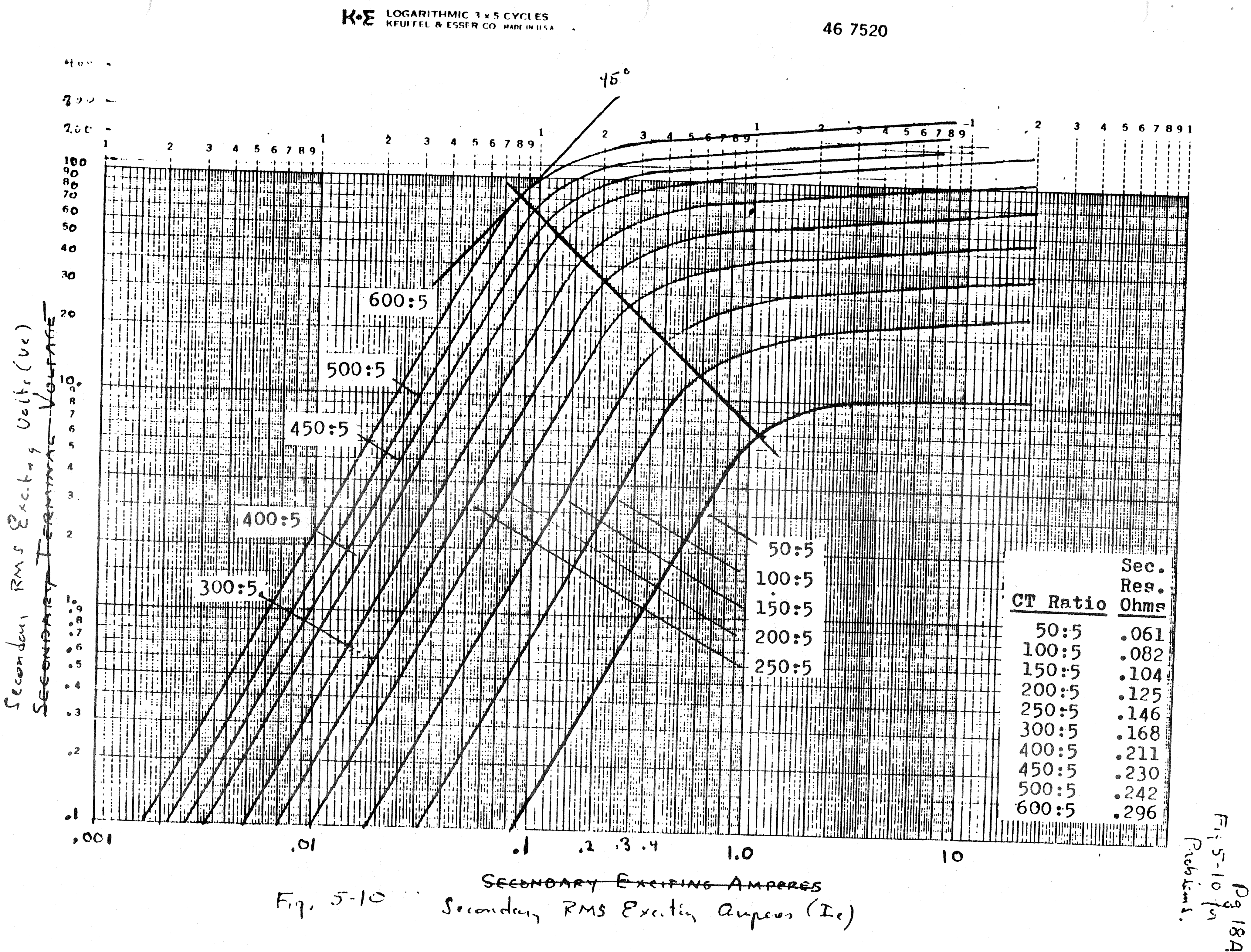
$I_P = 20 \times 5.23 = 104.6$ amps Primary to just operate The relay.

(b) from fig 5-10 The maximum V_S for 100:5 at the Saturation line is 12.8 volts where $I_e = 0.58$ amp.

$$I_S = \frac{V_S}{Z_B} = \frac{12.8}{0.59} = 21.69 \text{ amps.}$$

$I_P = 20 \times 21.69 = 433.90$ amperes Primary.

Pg 18A Fig 5-10 for Problems.



CT Ratio	Sec. Res. Ohms
50:5	.061
100:5	.082
150:5	.104
200:5	.125
250:5	.146
300:5	.168
400:5	.211
450:5	.230
500:5	.242
600:5	.296

Fig. 5-10

Prob. 5.1 Cont'd:

(PS1)

Part (c) Using 200:5 = 40:1 ; Load = $\frac{80}{40}$ = 2.0 choose Tap 2.5

when relay burden = $\frac{3.2}{2.5^2}$ = 0.512 ohm

CT Sec = 0.125 ohm

Lead burden = 0.380 ohm

Total Z_B = 1.017 ohms.

$V_s = I_s Z_B = 2.5 \times 1.017 = 2.54$ volts when $I_e = 0.64$ amps.

assuming direct addition! $I'_s = 2.5 + .064 = 2.564$ amps.

$I_p = 40 \times 2.564 = 102.56$ amps. to just operate relay.

an improvement over part (a) of 100:5 ratio & Tap 5.

(d) From fig 5-10; max. V_s for the 200:5 Tap to saturation is 26.5 volts, where $I_e = 0.28$ amp.

$I_s = \frac{V_s}{Z_B} = \frac{26.5}{1.017} = 26.06$ amps.

$I_p = 40 \times 26.06 = 1042.28$ amperes (better than part b)

(e) The use of a higher CT ratio & lower relay Tap is preferable.

Prob 5.2 pg 507:

$V_e = \frac{VA\ Burden}{I} = \frac{285}{10} = 28.5$ volts

From Fig 5-10 curve, minimum ratio would be 250:5 ratio.

Prob 5.3 pg 507: fig. 5.7 (not 5.8) 800:5 = 160:1

For the 2.0 ohm burden! $\frac{15,200}{800} = 19$ Times

from B2 Curve fig. 5-7 I_s = approx 18.6 Times.

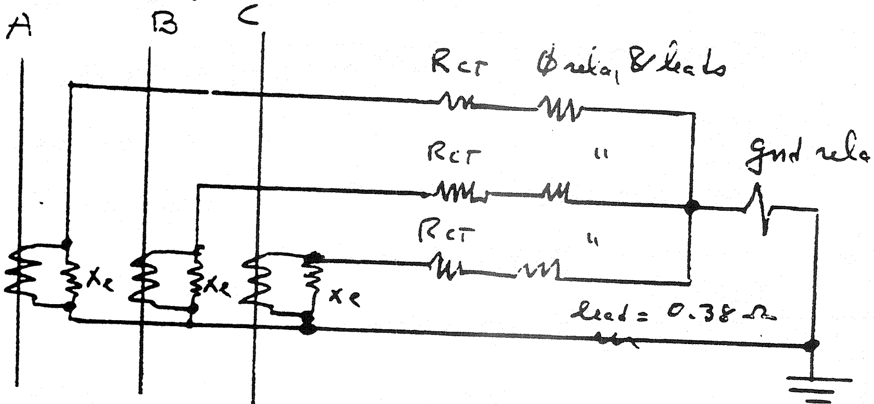
% error = $\frac{19-18.6}{19} 100 = 2.11\%$

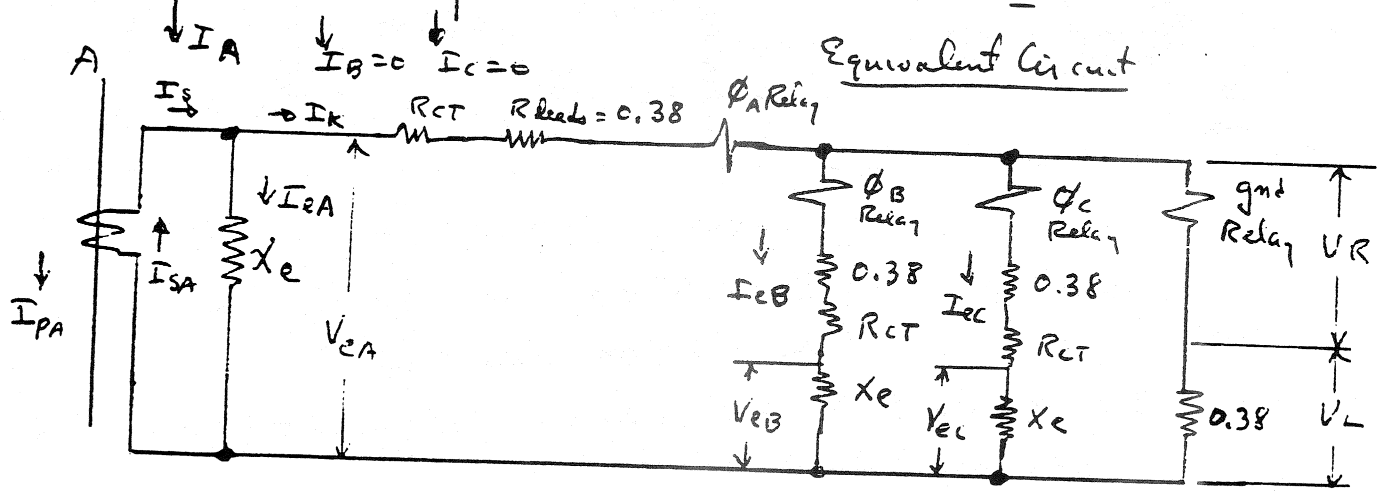
For the 4.0 ohm burden:

from B4 Curve fig. 5-7 I_s = approx 14.0 Times

% error = $\frac{19-14}{19} 100$ = 26.32% error.

Prob. 5.4 pg 507-508





Thus for each phase B and C:

$$I_e(\phi_{relay\ ohms} + R_{CT} + 0.38) + V_e = V_R + V_L$$

or $V_e = V_R + V_L - (\phi_{relay\ ohms} + R_{CT} + 0.38)\, I_e$

I_e & V_e are unknown but related, so a "cut & Try" method of Solution can be used. First assume $V_e = V_R + V_L$ and determine I_e. Then reduce V_e by $(\phi_{relay\ ohms} + R_{CT} + 0.38)\, I_e$ and redetermine a new Value for I_e, etc. Usually one or two Tries will be sufficient.

With Various CT Taps, the phase relay burdens are:

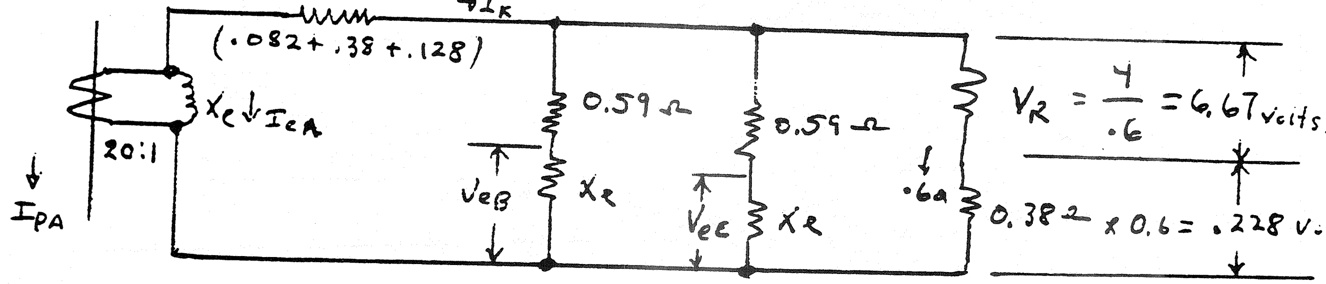
50:5 Tap (10:1) $\frac{80}{10} = 8.0$ amp sec. Use Tap 10 where $Z_\phi = \frac{3.2}{10^2} = 0.32$

100:5 Tap (20:1) $\frac{80}{20} = 4.0$ " " use Tap 5 when $Z_\phi = \frac{3.2}{5^2} = 0.128$

150:5 Tap (30:1) $\frac{80}{30} = 2.67$ " " use Tap 3.5 when $Z_\phi = \frac{3.2}{3.5^2} = 0.261$

200:5 Tap (40:1) $\frac{80}{40} = 2.0$ " " Use Tap 2.5 when $Z_\phi = \frac{3.2}{2.5^2} = 0.512$

Prob. 5.4 Cont'd. a number of combinations of CT ratios and relay Taps may be selected of which one will provide the lowest primary current value to just operate the ground relays. As an example consider relay tap 0.6 and CT Tap 100:5, when $Z_\phi = 0.128$ ohm, $R_{CT} = .082$ ohm



$V_R + V_L = 6.895$ volts; First Try assume $V_{eB} = V_{eC} = 6.9$ volts where from curve $I_e = .35$ amp. $.35 \times .59 = .207$ v $V_{eB} = V_{eC} = 6.895 - .207 = 6.689$ volts

" " " $I_e = .345$ amps close enough.

$I_K = 0.6 + 0.345 + 0.345 = 1.29$ amp. $V_{eA} = 6.895 + 1.29 \times .59 = 7.66$ volts where from curve $I_{eA} = .37$ amp.

Direct adding $I_S = 1.66$ amp. and $I_P = 1.66 \times 20 = 33.2$ amps. Prim. Pick-up.

Compared to $20 \times .6 = 12$ amp Primary pick-up if I_e's are neglected.

Prob. 5.5 page 508. use Figure P5.5 pg 508:

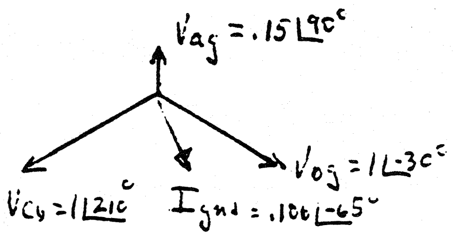
Ground Burden = 15 VA, 120 v. at 25° lead: $Z_g = \frac{120^2}{15\angle 25°} = 960\angle -25°$ ohms.

$3V_0$ for the fault $= V_{ag} + V_{bg} + V_{cg} = .15\angle 90° + 1.0\angle -30° + 1\angle 210°$

$= j.15 + .866 - j.50 - .866 - j.5 = .85\angle -90°$ percent

$= 120 \times .85 = 102\angle -90°$ volts.

$I_{gnd} = \frac{102\angle -90°}{960\angle -25°} = 0.10625\angle -65°$ ampere Secondary.



Residual Burden Distribution

Phase a: $VA = I_{gnd} V_{ag} = .106 \times .15 \times 120 = 1.9$ VA, I lags V 155°

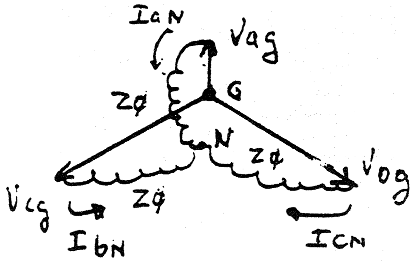
Phase b: $VA = I_{gn} V_{bg} = .106 \times 120 = 12.75$ VA I lags V 35°

Phase c: $VA = I_{gn} V_{cg} = .106 \times 120 = 12.75$ VA I leads V

(cont'd)

Prob. 5.5 Cont'd.

Phase Burden = 25 VA, 69.5 volts 0°; $Z_\phi = \frac{69.5^2}{25} = 193.21 \angle 0°$ ohms line to neutral.



$V_{NG} = V_0 = \frac{102}{3} = -j34$ volts from above & eq 7.8 Page 205.

$V_{\phi N} = V_{\phi G} - V_{NG} = V_{\phi G} + j34$ volts secondary

$= V_{\phi G} + j\,0.283$ per unit

$V_{aN} = .15\angle 90° + .283\angle 90° = .433\angle 90°$ per unit $= .433 \times 69.5 = 30.12$ volts sec.

$I_{an} = \frac{30.12\angle 90°}{193.21} = .157\angle 90°$ amps.

a phase burden $= V_{ag} I_{an} = .15 \times 69.5 \times .157 = $ 1.64 VA, 0° angle.

$V_{bN} = 1\angle -30° + .283\angle 90° = .866 - j.5 + j.283 = .866 - j.217 = .893\angle -14.07°$ pu $= 62.06\angle -14.07$ volts sec.

$I_{bN} = \frac{62.06}{193.2} = .321\angle -14.07°$ amp.

b phase burden $= V_{bg} I_{bN} = 69.5 \times .321 = $ 22.33 VA

I_{bN} / V_{bG}: I lead, 15.93°

$V_{cN} = 1\angle 210 + .283\angle 90° = -.866 - j.5 + j.283$

$= -.866 - j.217 = .893\angle -165.93°$ pu $= 62.06\angle -165.93°$ volts.

$I_{cN} = \frac{62.06}{193.2} = .321\angle -165.93°$ amp.

c phase burden $= V_{cg} I_{cN} = 69.5 \times .321 = $ 22.33 VA

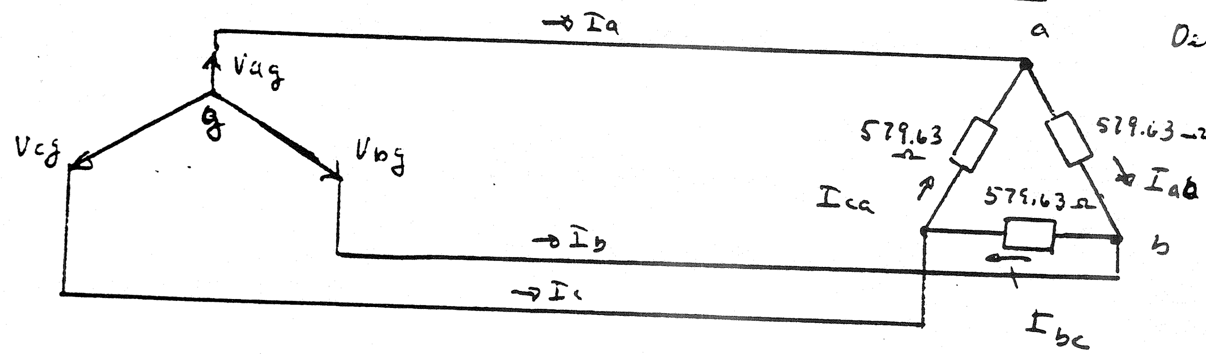
I_{cN} / V_{cG}: I lags 15.93°

Total Burden: Ground & phase burdens should be added phasorally - direct addition usually ok unless values are near the capacity of the VT.

	Direct addition		Phasor Addition
a	gnd 1.9 $\angle -155°$	gnd	$-1.72 - j.803$
	φ 1.64 $\angle 0°$	φ	1.64
	3.54 VA		$.08 - j.803 = .81$ VA
b	gnd 12.75 $\angle -35°$	gnd	$10.44 - j7.31$
	φ 22.33 $\angle 15.93$	φ	$21.47 + j6.13$
	35.08 VA		$31.91 - j1.18 = 31.93$ VA
c)	gnd 12.75 $\angle 85°$	gnd	$1.11 + j12.70$
	φ 22.33 $\angle -15.93$	φ	$21.47 - j6.13$
	35.08 VA		$22.58 + j6.57 = 23.52$ VA

50 VA Transformer is adequate.

Prob. 5.5 Alternate Solution for the Phase burdens.



Delta burden is $3 \times 193.2 = 579.6$ ohms

$V_{ag} = .15 \times 69.5 = j\,10.43$ volts.

$V_{bg} = 69.5\angle{-30^\circ}$ volts $= 60.19 - j\,34.75$

$V_{cg} = 69.5\angle{210^\circ}$ volts. $= -60.19 - j\,34.75$

$V_{ab} = V_{ag} - V_{bg} = 75.26\angle{143.11^\circ}$ volts

$V_{bc} = V_{bg} - V_{cg} = 120.38\angle{0^\circ}$ volts.

$V_{ca} = V_{cg} - V_{ag} = 75.26\angle{-143.11^\circ}$ volts.

$I_{ab} = \dfrac{75.26}{579.63} = .1298\angle{141.11^\circ}$ amp

$I_{bc} = \dfrac{120.38}{579.63} = .2077\angle{0^\circ}$ amp

$I_{ca} = \dfrac{75.26}{579.63} = .1298\angle{-141.11^\circ}$ amp.

$I_a = I_{ab} - I_{ca} = .157\angle{90^\circ}$ amps

$I_b = I_{bc} - I_{ab} = .321\angle{-14.1^\circ}$ amps

$I_c = I_{ca} - I_{bc} = .321\angle{-165.9^\circ}$ amps

} checks previous calculations.

Prob. 7.1 page 509.

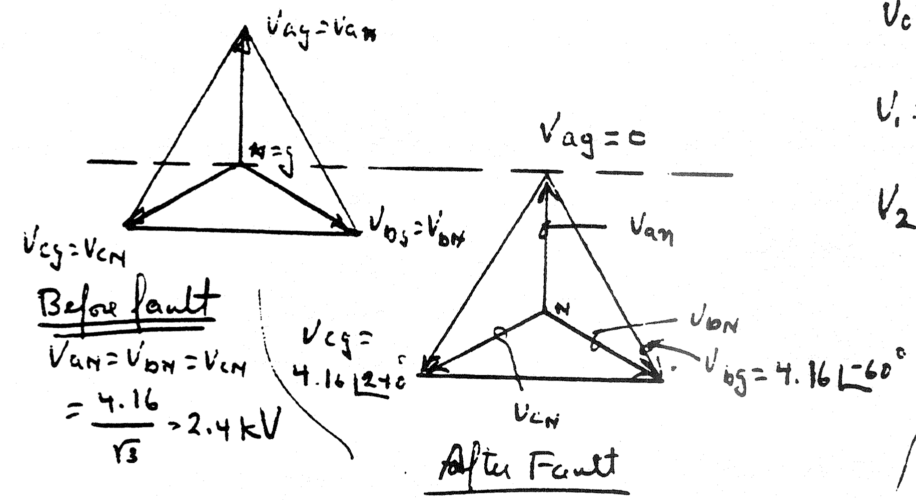


Fig. 7-1 pg 201

$V_0 = \frac{1}{3}(V_{ag} + V_{bg} + V_{cg}) = -j\,2.40$ kV.

$V_1 = \frac{1}{3}(V_{ag} + aV_{bg} + a^2V_{cg}) = +j\,2.40$ kV

$V_2 = \frac{1}{3}(V_{ag} + a^2V_{bg} + aV_{cg}) = 0$

For a phase-a-to ground Fault:

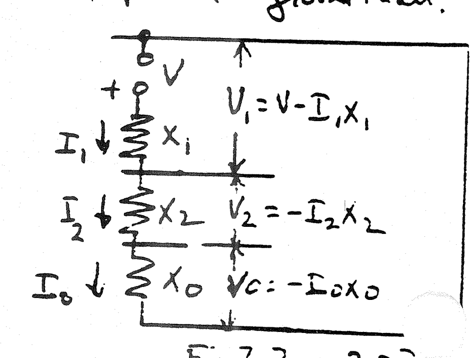


Fig 7 3 pg 203.

$X_1 = X_2$ approach 0, X_0 approaches ∞ So answers above for V_1, V_2, V_0 are for limit case. Currents in unfaulted phases mean that V_{bg} & V_{cg} during fault are slightly less & slightly different angle, so there will be a very small V_2, and V_1 less than V, etc.

Problem 7, 2 page 509

1. $X_{co} = \frac{1}{2\pi f c} = \frac{10^6}{2\pi f C} = \frac{10^6}{2\pi \times 60 \times 0.4} = 6631.46$ oh[ms]

I charging to ground $= \frac{V_{LN}}{-jX_c} = \frac{\frac{4160}{\sqrt{3}}}{6631.46} = .362$ amps per ph[ase] at 4.16kV.

2 $X_1 = X_2 = 0+$

$X_0 = -j\,6631.46\ \Omega$ or $\frac{100}{4.16^2} \times 6631 = 38,319.7$ pu on 100MVA

$I_F = 3I_0 = \frac{3V}{X_1+X_2+X_0} = \frac{3 \times \frac{4160}{\sqrt{3}}}{6631.46} = 1.087\angle 90^\circ$ amps 4.16k[V]

3. $100/5 = 20$ $20 \times 0.5 = 10$ amperes primary

Neither this nor the 5a primary pick up relay can operate

4. $\frac{2.4}{10}(5000) = 1200$ KVA

$X_{22} = 6.67\%$ on 1200KVA

$Z_{Res} = (29.2 + j\,12.4)\%$ on 1200 KVA

5. X_0 Total $= .0667 + 3(.124) = .439$ pu.

X_1 Total $= .024$ pu

$\frac{X_0}{X_1} = \frac{.439}{.024} = 18.[3]$ which < 20 ok

all on 1200KVA

$R_0 = 3(.292) = .876$ pu.

$X_0 = .439$ pu

$\frac{R_0}{X_0} = \frac{.876}{.439} = 2.0$ just ok.

6. $X_1 = X_2 = .024$ pu

$X_0 = .876 + j\,.439$ pu

$I_1 = I_2 = I_0 = \frac{1.0}{.876 + j.439 + j.024 + j.024} = \frac{1.0}{.876 + j.487}$

$= \frac{1.0}{1.002\angle 29.07} = .998\angle{-29.07^\circ}$ pu.

$I_{GF} = 3I_0 = 2.993\angle{-29.07^\circ}$ pu $= 2.993 \times \frac{1200}{\sqrt{3} \times 4.16} = 498.5$ amps at 4.1[6kV]

$.292 \times \frac{(4.16)^2}{1.2} = 4.21\Omega$

$.292 + j.124 = .317\angle 23^\circ$

$\frac{.292}{.317} = .92$

7. Zig Zag grounding Transf 1 min rated, 170 amps per phase 4160 V $X = 6.67$ on 120[0KVA]

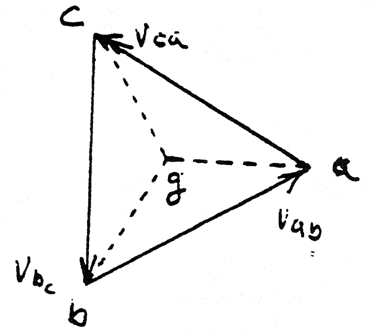
Resistor Stainless steel, 10 sec., 500 amps; $4.21\ \Omega$ at .92PF.

8 Relay $\frac{498.5}{100/5} = 24.93$ amps sec. ok for 0.5 amps pickup: 49.85 x

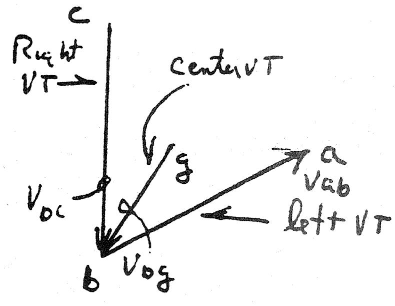
Problem 7.3 page 509 - 510 (Fig. P7.3 Correct by Connecting the right lead of the middle VT secondary to ground. Add a connection dot just left of the ground connection at the aux. VT's.) (Ps 2

a) Voltage check

Assume that the bus phasors for balanced conditions are: Voltage phasors

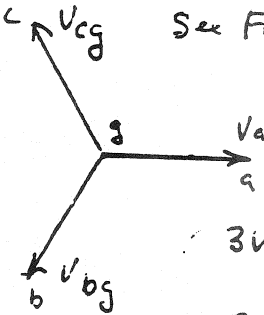


System Phasors.



VT Primary & Secondary.

See Fig 3.3b pg 57



Aux. VT Primary

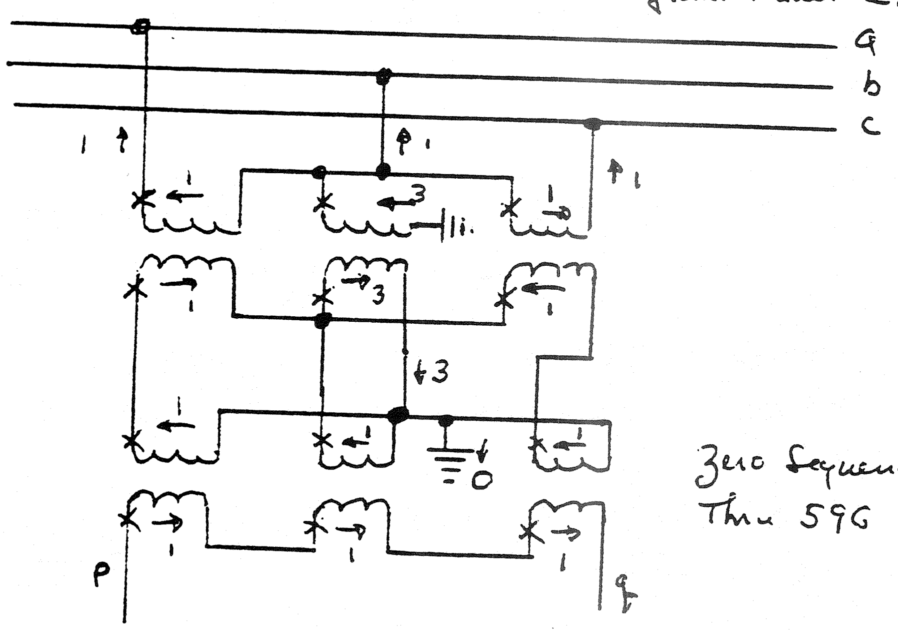
aux. VT Sec

$3V_0 = V_{pq} = V_{ag} + V_{bg} + V_{cg}$

$= 0$ for balanced cond.

$3V_0$ finite for gnd faults.

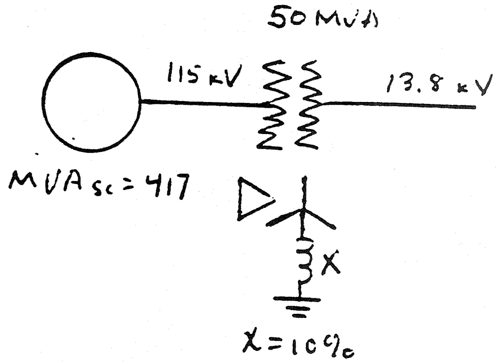
b) Zero sequence current check: For a ground Fault I_0 flows in each phase:



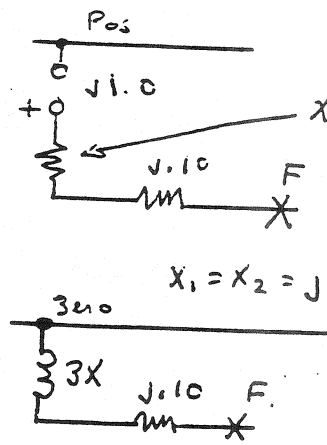
Zero Sequence Current can flow Thru 59G

Prob. 7.4 page 511

(PS 26



On 50 MVA Base



$X_1 = \frac{50}{417} = j.12$ pu eq A4.1-- pg 138

$X_1 = X_2 = j.22$ pu.

$X_0 = j.10 + 3X$

a) For a 13.8 kV ground fault $I_a = 4000$ amps

$I_{Base} = \frac{50{,}000}{\sqrt{3}\ 13.8} = 2091.85$ amps at 13.8 kV.

$I_a = I_1 + I_2 + I_0 = \frac{4000}{2091.85} = 1.91$ per unit.

$I_1 = I_2 = I_0 = \frac{1.91}{3} = \frac{j1.0}{j.22 + j.22 + j.10 + 3X} = .64$ pu.

$.64(.54 + 3X) = 1.0$; $1.91X = 1.0 - .34$;

$X = 0.34$ pu $= \frac{13.8^2 \times .34}{50} = 1.31$ ohms.

b) For a 13.8 kV solid gnd. fault (X=0)

$I_1 = I_2 = I_0 = \frac{j1.0}{j.22 + j.22 + j.10} = \frac{1}{.54} = 1.85$ pu.

$I_a = 3 \times 1.85 = 5.56$ pu $= 5.56 \times 2091.85 = 11{,}621.38$ amps.

$\% = \frac{4000}{11{,}621.38} 100 = \frac{1.91}{5.56} 100 = 34.4\%$

aside) $I_{3\phi\ Fault} = \frac{1}{.22} = 4.55$ pu. Solid gnd fault is $\frac{5.56}{4.55} 100 = 122.3\%$ larger.

c) Part a) but with a resistor instead of a reactor

$I_1 = I_2 = I_0 = .64 = \frac{1.0}{j.22 + j.22 + j.10 + 3R}$ (more convenient to use $V = 1.0\angle 0°$?)

$.64(j.54 + 3R) = 1.0$; $1.91R + j.344 = 1.0$

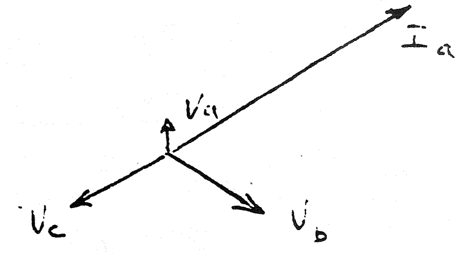
Squaring each Term: $(1.91R)^2 + (.344)^2 = (1.0)^2$

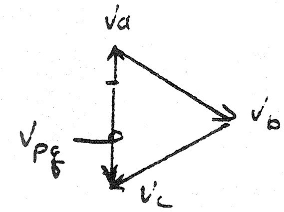
$R = \sqrt{\frac{1.0 - .118}{3.66}} = \sqrt{.241} = .491$ pu $= .491 \times \frac{13.8^2}{50} = 3.81$ ohms

Check: R = .491 pu, 3R = 1.47: $I_1 = I_2 = I_0 = \frac{1.0}{1.47 + j.54} = \frac{1.0}{1.57\angle 20.17°} = .64\angle -20.17°$ pu.

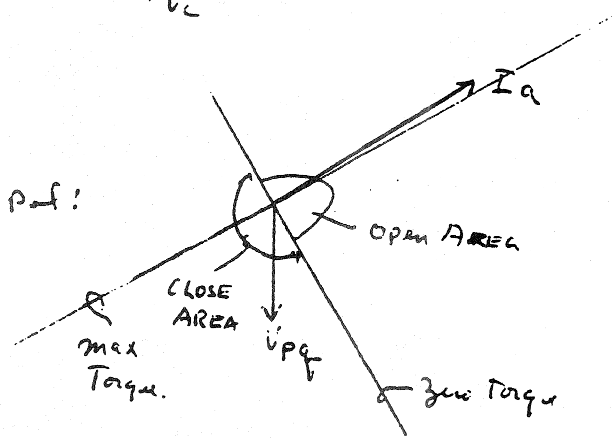
Prob 7.5 Pages 511-12

a)





For a Phase-a-to-ground Fault.

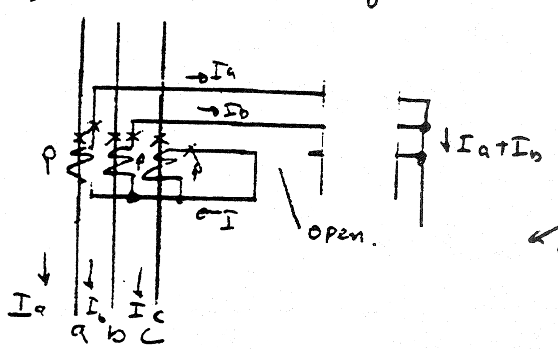


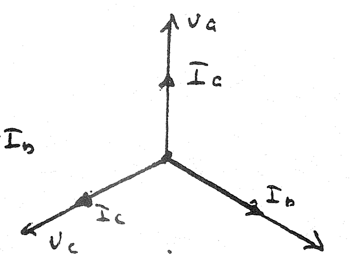
Current & Voltage on relay pol. to non pol!

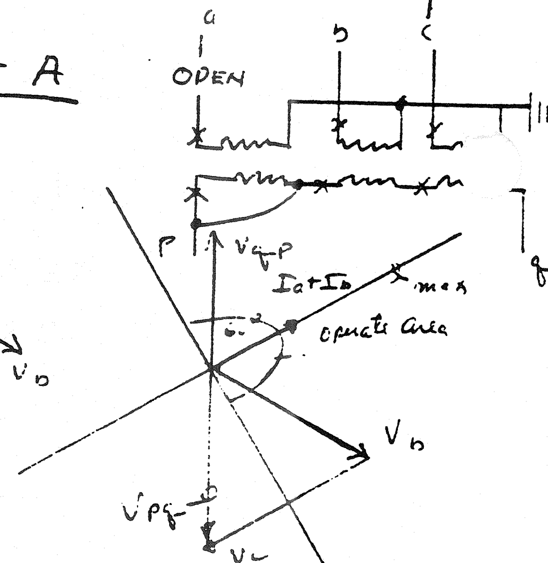
Connections Not Correct:

Either current or voltage must be reversed on Relay

b) Suppose voltage is reversed! **TEST A**





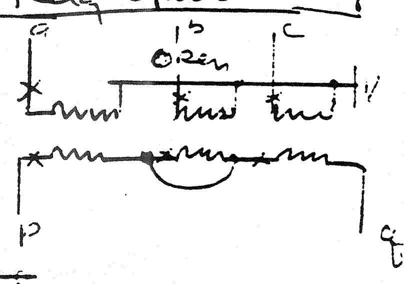


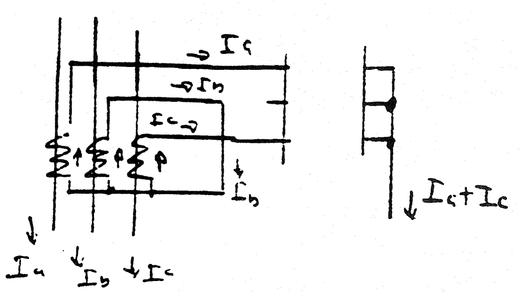
Phase C CT shorted & Sec. Opened,
phase a VT opened & Sec shorted

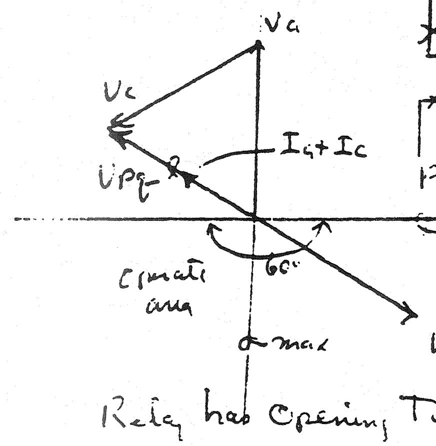
TEST B

Phase b CT shorted & Sec. opened
Phase c VT opened & Sec. shorted

Relay operates





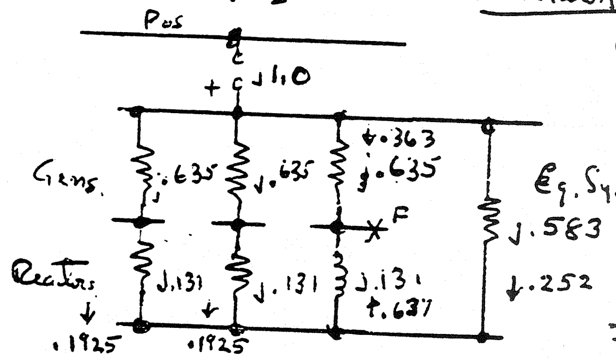


Both Test Valid

Relay has Opening Torque.

note: either combination can be selected for variations

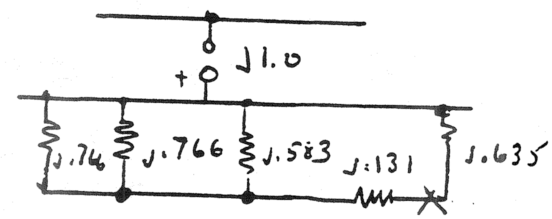
Prob. 8.1 page 513 Generators are 13.8 kV. on 100 MVA Base.



each gen $X_d'' = \frac{100}{21.875} \times .139 = .635$ pu.

Reactor $X = \frac{100}{13.8^2} \times .25 = .131$ pu.

=



.396 .604

$$\frac{\frac{.766}{2} \times .583}{.966} = .231 \text{ pu.}$$

.131

.362

.363 .637

$$X_1 = \frac{.362 \times .635}{.997} = j.231 \text{ pu.}$$

a) $I_{3\phi}$ at F $= \frac{j1.0}{j.231} = 4.329$ po $= 4.329 \times \frac{100{,}000}{\sqrt{3}\ 13.8} = 18{,}111.25$ amps at 13.8 kV

Current thru faulted generator to fault $= 4.329 \times .363 = 1.571$ pu $= 6572.59$ amp. at 13.8 kV

Current thru other two generators to fault $= 4.329 \times .1925 = .833$ pu $= 3485.02$ amp. at 13.8 kV

---

b) $I_{rated} = \frac{21{,}875}{\sqrt{3} \times 13.8} = 915.18$ amperes at 13.8 kV.

choose 1000/5 CT's = 200 so $I_{rated} = \frac{915.15}{200} = 4.58$ a. sec.

If the fault is in the generator differential zone, the differential relay will receive $\frac{18{,}111.25}{200} = 90.56$ amp. sec. ; $\frac{90.56}{.14} = 646.8$ times pick-up

---

c) For a line-to-gnd fault at F, from a) $X_1 = X_2 = j.231$ pu.

$Z_0 = 11 + j4.73$ pu. (X_0 system ∞ since not given)

$$I_1 = I_2 = I_0 = \frac{j1.0}{j.231 + j.231 + 11 + j4.73} = \frac{j1.0}{11 + j5.192} = \frac{1.0\angle 90^\circ}{12.164\angle 25.27^\circ} = .082\angle 64.73^\circ$$ pu unit

$I_a = 3I_0 = .247\angle 64.73^\circ$ pu $= 1033.37$ amp. at 13.8 kV.

$= \frac{1033.37}{200} = 5.17$ amp. sec. 36 times relay pick-up.

Problem 8.2 pg. 514. ON 100 MVA Base.

Total Capacitance $= .24 + .25 + .004 + .03 + .004 + .0005 = 0.52850$ mfd.

$$X_c = -j\frac{1}{2\pi f C} = -\frac{10^6}{j\,2\pi\,60 \times .5285} = -j5019.077 \text{ ohms per phase}$$

$$= \frac{100 \times 5019.077}{18^2} = -j\,1549.098 \text{ per unit.}$$

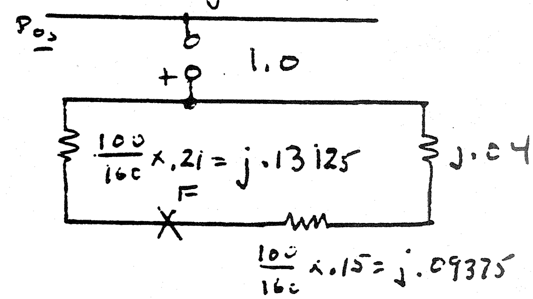
$R = 64.14$ KW at 138 volts: $I = \frac{\text{watts}}{V} = \frac{64,140}{138} = 464.782$ amps.

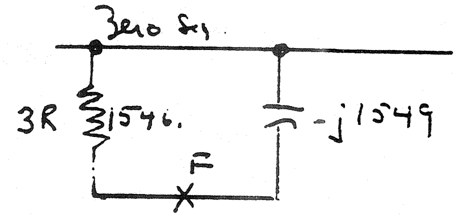
$$= \frac{V}{I} = \frac{138}{464.782} = 0.2969 \text{ ohm.} = .2969\left(\frac{18,000}{240}\right)^2$$

$$= 1670.136 \text{ ohm at } 18\text{ kV}.$$

$$3R = 5010.4069 \text{ ohms} = 5010\,\frac{100}{18^2} = 1546.422 \text{ per unit.}$$

a) Phase a to ground fault:





$$X_1 = X_2 = \frac{.13125 \times .13375}{.265} = .06624 \text{ pu.}$$

(.49528 .50472)

$$Z_0 = \frac{1546 \times .1549\angle{-90}}{1546 - j.1549}$$

$$= \frac{1546 \times .1549\angle{-90}}{2188.86\angle{-45.05^\circ}}$$

(.70650∠45.05 .70772∠−44.75°)

$$= 1094.43\angle{-44.95^\circ} \text{ pu.}$$

$$I_1 = I_2 = I_0 = \frac{1.0}{1094.4} = 0.00091\angle{44.95^\circ} \text{ pu.}$$

$$3I_0 = 0.00274\angle{44.95^\circ} \text{ pu} = \frac{100,000}{\sqrt{3}\,18} \times .00274 = 8.79 \text{ amperes at } 18\text{kV}.$$

---

b) Three phase Fault

$$I_{a3\phi} = \frac{1.0}{j.06624} = 15.097 \text{ pu}$$

$$= 15.097 \times \frac{100,000}{\sqrt{3}\,18} = 48,422.43 \text{ amps at } 18\text{ kV}.$$

c) $I_{rated} = \frac{160,000}{\sqrt{3}\ 18} = 5132$ amperes at 18KV.

use either 5000:5 (1000:1) OR 6000:5 (1200:1)

$I_{3\phi} = \frac{48,422}{1000} = 48.42$ amp-sec. $\left(\frac{48.42}{.14} = 345.9 \times \text{pickup}\right)$ OR $\frac{48,422}{1200} = 40.35$a. & $\left(\frac{40.35}{.14}\right) = 288 \times$ pick up.

$I_{ag.} = \frac{8.79}{1000} = .00879$ a. Relay will Not Operate on Ground Faults.

d) $3I_o$ current in the neutral and resistor is

$= 8.79 \times .70772 = 6.22$ amperes.

Thus the resistor $I = 6.22 \frac{18,000}{240} = 466.56$ amperes sec.

Voltage across resistor $466.56 \times 0.2969 = 138.52$ volts,

$\frac{138.52}{5.4} = 25.65$ times relay pick-up.

---

e) From above $I_{sec} = 466.56$ amperes. CT should be selected considering the short time current ratings of the CT & relay 50/51. One possibility is 100:5 & 1.0 ampere pick-up. Both relays should not operate on 345 kV ground faults reflected thru the power transformer winding capacitance (Pg. 259 Sect 8.7.2).

Prob. 8.3 page 515.

a & b: from Fig P8.3 pg 515 measure the KVA total at different angles from 0 to the point on the curve. For the 50 MVA 13.2 kV unit the curve per unit is 50 MVA = 1 pu.

in per unit: $Z_{perunit} = \frac{KV^2_{pu}}{MVA_{pu}} = \frac{1}{MVA_{pu}}$ since KV = 1.0 pu.

in primary ohms: $Z_{ohms} = \frac{KV^2}{MVA}$; in secondary ohms: $Z_\Omega = \frac{KV^2 R_c}{MVA\, R_V}$ eq. 8.15 Pg. 265.

$R_c = 3000:5$, $R_V = 13,200/110$

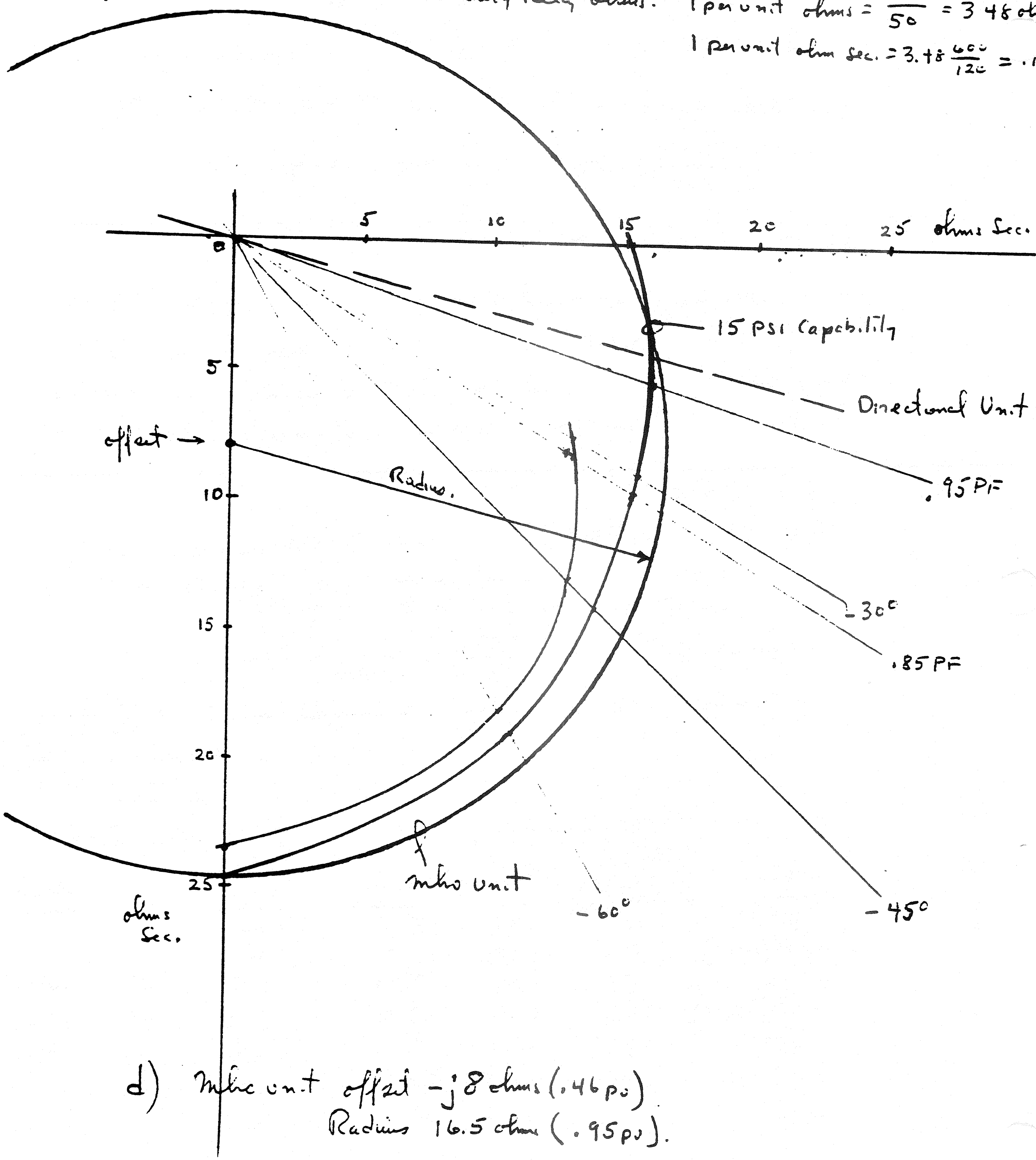
$R_c = 600$ $R_v = 120$

Angle.	15 PSI Capability MVA pu from fig P.8.3	Z_{pu}	Z ohms Sec.	Steady State Stability MVA pu from fig P8.3	Z_{pu}	Z ohms Sec.
0°	1.15	.87	15.15	—		
−18.19° (.95PF)	1.025	.976	17.0	—		
−30°	.97	1.03	17.96	1.15	.87	15.15
−31.79° (.85PF)	.96	1.04	18.15	1.12	.89	15.56
−45°	.875	1.14	19.91	.96	1.04	18.15
−60°	.80	1.25	21.78	.84	1.19	20.74
−90°	.705	1.42	24.71	.74	1.35	23.55

example: $Z_{pu} = \frac{1}{1.025} = .976$ pu, $Z_{ohms\,sec} = \frac{13.2^2 \times 600}{50 \times 120} \times .976 = 17.0$ ohms sec.

c) Curve plotted in secondary Relay ohms. 1 per unit ohms $= \frac{13.2^2}{50} = 3.48$ ohms

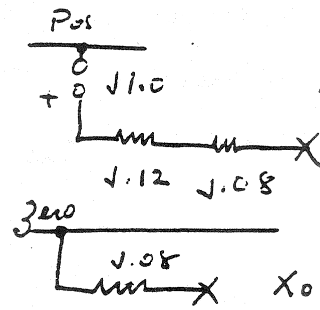
1 per unit ohm sec. $= 3.48 \frac{600}{120} = 17.4$



d) Mho unit offset $-j8$ ohms (.46 pu)
Radius 16.5 ohms (.95 pu).

Prob. 9.1 pg. 516 per unit on 50MVA.

a) 69kV Phase-a-to-gnd fault.



$X_1 = X_2 = j.20$ pu.

$X_0 = j.08$ pu.

$$I_1 = I_2 = I_0 = \frac{j1.0}{j.20 + j.20 + j.08} = \frac{j1.0}{j.48} = 2.08 \text{ pu.}$$

$$I_{aF} = 3I_0 = 6.25 \text{ pu} = \frac{50,000}{\sqrt{3}\,69} = 2614.81 \text{ amps at } 69kV.$$

$$I_{bF} = I_{cF} = 0$$

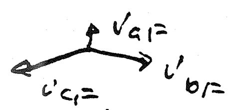
b)

$$V_{1F} = j1.0 - j.20 \times 2.08 = j.584 \text{ pu.}$$
$$V_{2F} = 0 - j.20 \times 2.08 = -j.417 \text{ pu}$$
$$V_{0F} = 0 - j.08 \times 2.08 \quad -j.167 \text{ pu}$$

$$V_{aF} = V_{1F} + V_{2F} + V_{0F} = 0$$

$$V_{bF} = a^2 V_{1F} + aV_{2F} + V_{0F} = .584 \angle \overset{-30^\circ}{90+240^\circ} + .417 \angle \overset{30^\circ}{-90+120^\circ} - j.167$$
$$= .5058 - j.2920 + .3611 + j.2085 - j.167$$
$$= .8669 - j.2505 = .9024 \angle -16.12^\circ \text{ pu} = 35,949.06 \text{ volts}$$

$$V_{cF} = aV_{1F} + a^2 V_{2F} + V_{0F} = .584 \angle \overset{210^\circ}{90+120} + .417 \angle \overset{150}{-90+240} - j.167$$
$$= -.5058 - j.2920 - .3611 + j.2085 - j.167$$
$$= -.8669 - j.2505 = .9024 \angle -163.88^\circ \text{ pu}$$



c) In passing thru the delta-wye bank. (pg 147-149)

$$I_{1A} = 2.08 \angle -30^\circ \text{ pu.} = 1.80 - j1.04$$
$$I_{2A} = 2.08 \angle +30^\circ \text{ pu.} = 1.80 + j1.04$$
$$I_{0A} = 0$$

$$I_A = I_{1A} + I_{2A} = 1.80 - j1.04 + 1.80 - j1.04 = 3.60 \angle 0^\circ \text{ pu.}$$

$$I_B = a^2 I_{1A} + aI_{2A} = 2.08 \angle \overset{210^\circ}{-30+240} + 2.08 \angle \overset{150}{30+120}$$
$$= -1.80 - j1.04 - 1.80 + j1.04 = 3.60 \angle 180^\circ \text{ pu}$$

$$I_C = aI_A + a^2 I_{2A} = 2.08 \angle \overset{90}{-30+120} + 2.08 \angle \overset{270}{30+240} = 0$$

d) Voltages are best calculated from the drops from the source remembering that for the 13.8 kV side $V_{gen} = 1\angle{90-30} = 1\angle{60^\circ}$

$V_1 = V_1 - I_1 X_1 = 1\angle{60^\circ} - 2.08\angle{-30} \times j\,.12$

$= .5 + j.866 - .1248 - j.2162 = .3752 + j.6498 = .7504\angle{60^\circ}$ pu.

$V_2 = 0 - I_2 X_2 = 0 - 2.08 \times .12\angle{30+90} = .1248 - j.2162 = .2496\angle{-60^\circ}$ pu

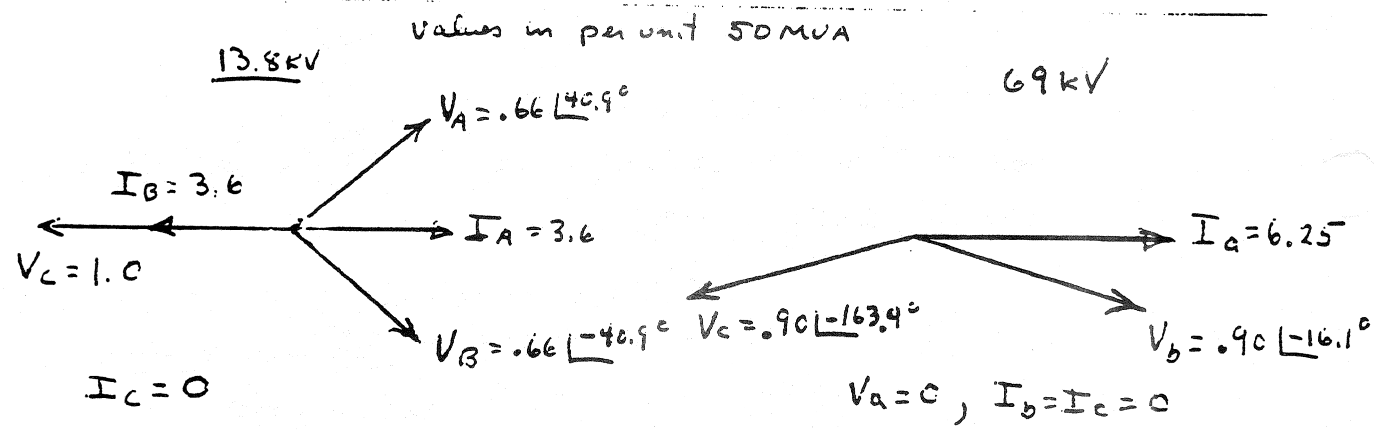
$V_0 = 0$

$V_A = V_1 + V_2 = .50 + j.4336 = .6618\angle{40.93^\circ}$ pu.

$V_B = a^2 V_1 + a V_2 = .7504\angle{60+240}^{-60^\circ} + .2496\angle{-60+120}^{+60^\circ} = .3752 - j.6498 + .1248 + j.2162$

$= .50 - j.4336 = .6618\angle{-40.93^\circ}$ pu.

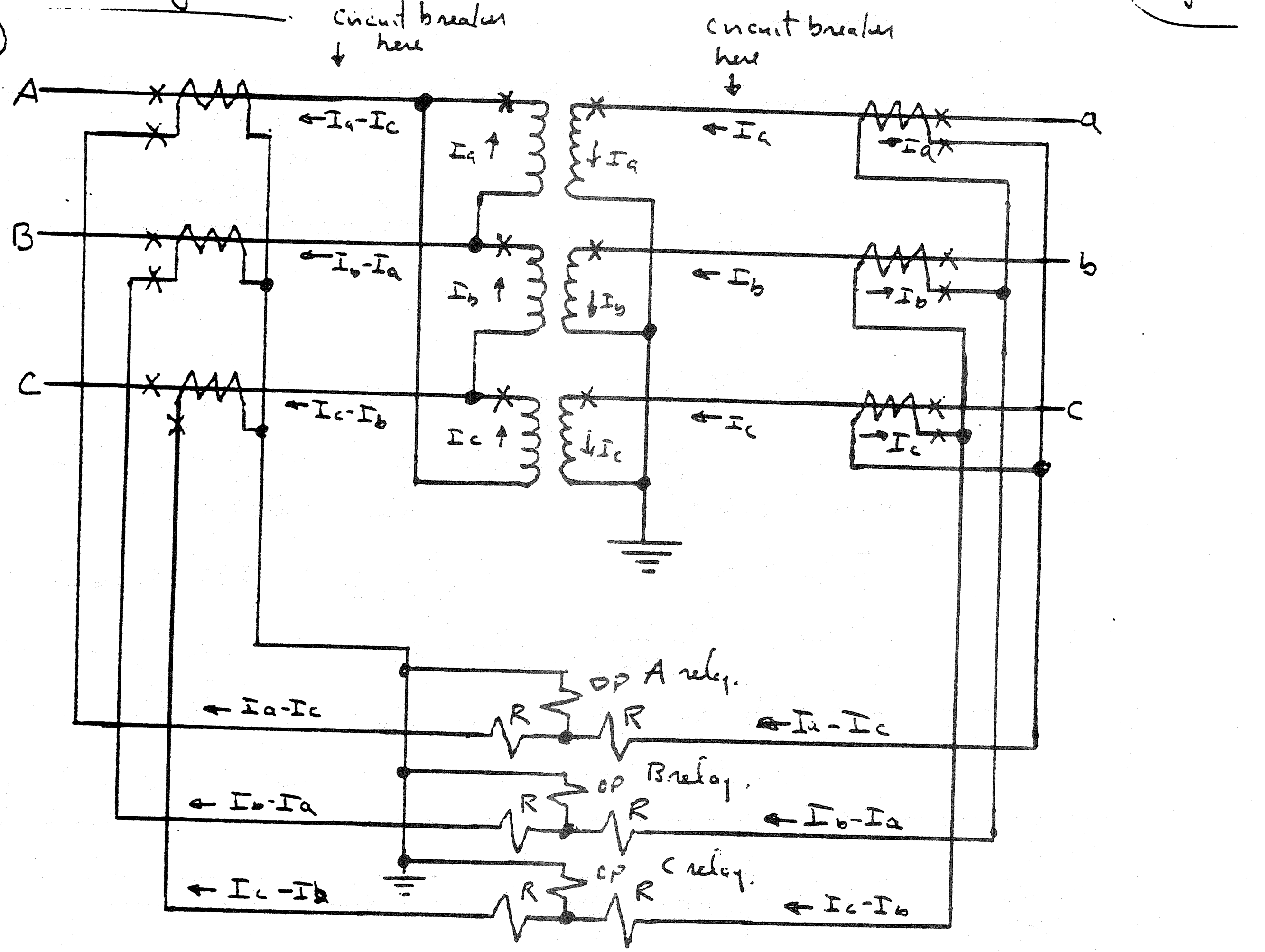
$V_C = a V_1 + a^2 V_2 = .7504\angle{60+120}^{180^\circ} + .2496\angle{-60+240}^{180^\circ} = 1.0\angle{180^\circ}$ pu.



The 69 kV phase a to ground fault looks like a 13.8 kV phase B-C Fault.

Prob. 9.2 pgs 516-517.

a)



I_a, I_b, I_c, etc are balanced currents used to connect differential relay:

b) $I_{69} = \frac{50,000}{\sqrt{3}\ 69} = 418.37$ amps at 69 kV

use 500:5 CT ratio: $I_{sec} = \frac{418.37}{100} = 4.18$ amps.; $I_{relay} = \sqrt{3} \times 4.18$
$= 7.25$ amps.

$I_{13.8} = \frac{50,000}{\sqrt{3}\ 13.8} = 2091.85$ amps at 13.8 kV.

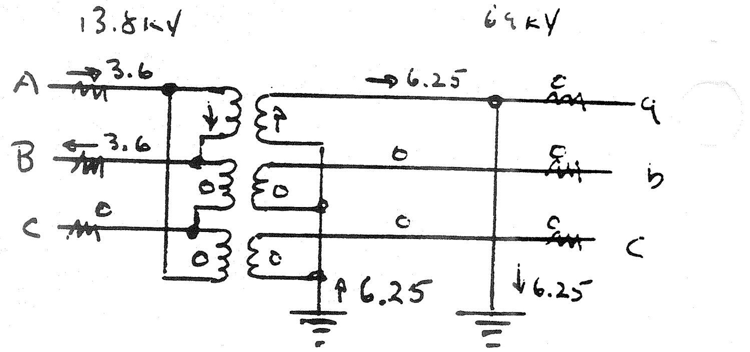
use 2200:5 CT ratio: $I_{sec} = \frac{2091.85}{440} = 4.75$ amps $= I_{relay}$

c) $\frac{I_{69}}{I_{13.8}} = \frac{7.25}{4.75} = 1.52$; nearest taps are 6 & 4 where $\frac{6}{4} = 1.5$

$M = \frac{1.52 - 1.5}{1.5} 100 = 1.61\%$ miss match, very good. (Eq. 9.1 Pg 286)

Prob. 9.2 Contd.

d) Per unit Currents from Prob. 9.1 a and c



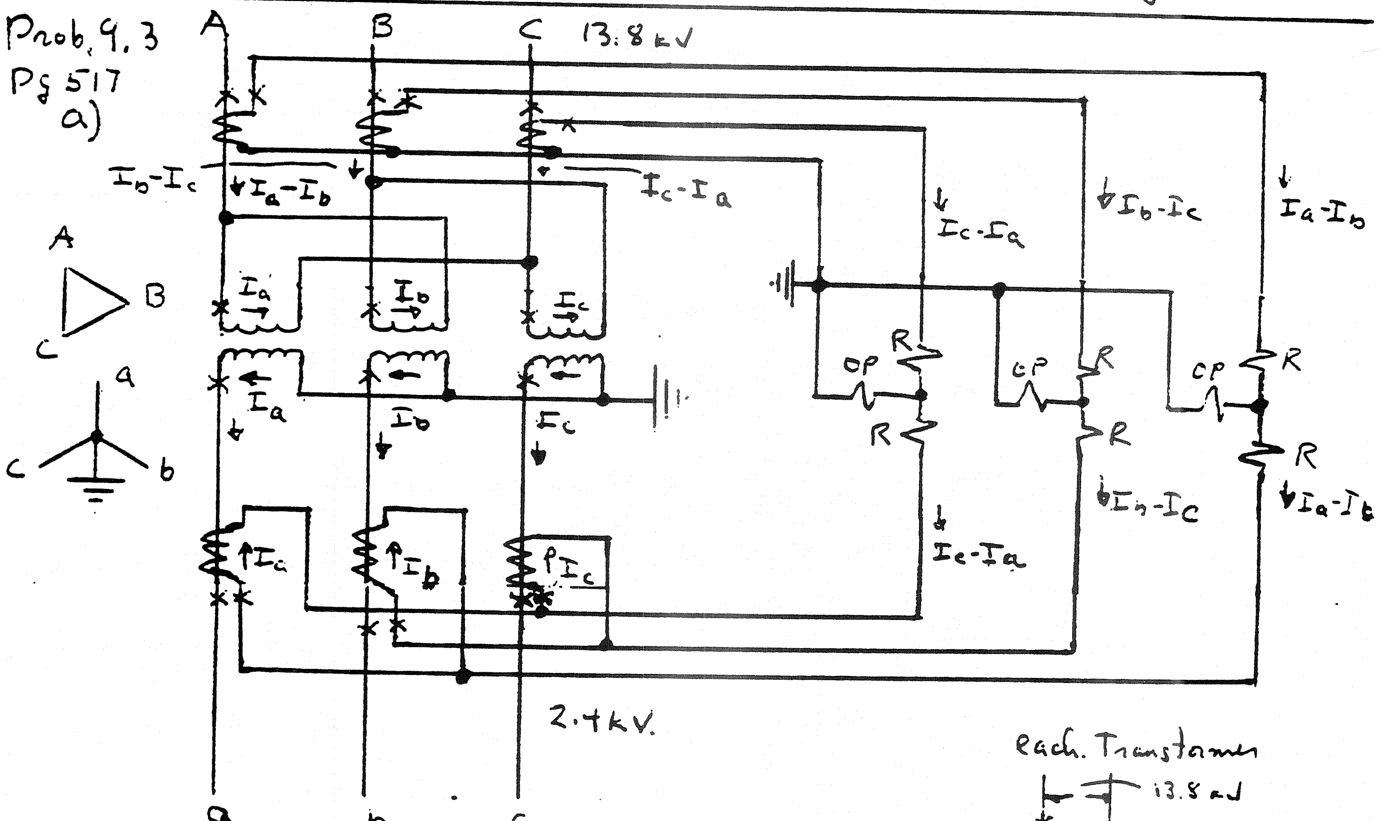
For the external 69 kV ground fault, current only on the 13.8 kV side can flow thru the Differential relays. Two relays A & B receive 3.6 pu in their operating coils (OP)

$$I_{\text{fault primary}} = 3.6 \times \frac{50{,}000}{\sqrt{3} \times 13.8} = 7530.66 \text{ amperes at } 13.8 \text{ kV}$$

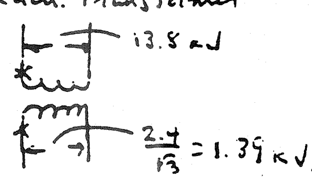
$$I_{\text{relay}} = \frac{7530.66}{440} = 17.12 \text{ ampere in relay.}$$

This should be ample to operate any transf. diff. relay. Note that the relays operate on the positive and negative sequence currents only for this ground fault.

Prob. 9.3
Pg 517
a)



Each Transformer



Prob. 9.3 Cont'd

b) $I_{13.8} = \frac{3000}{\sqrt{3}\ 13.8} = 125.51\,a.$ at 13.8 kV Select 150:5 (30)

$I_{sec} = I_{relay} = \frac{125.51}{30} = 4.18$ amps.

$I_{2.4} = \frac{3000}{\sqrt{3}\ 2.4} = 721.69\,a.$ at 2.4 kV Select 1000:5 (200)

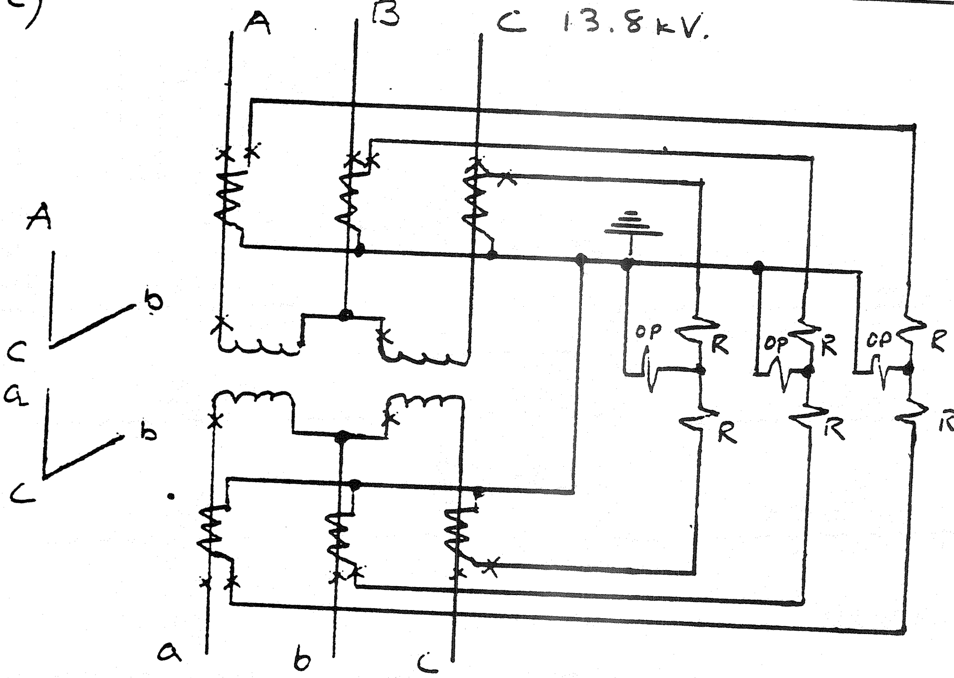
$I_{sec} = \frac{721.69}{200} = 3.61$ amps $I_{relay} = \sqrt{3}\,(3.61) = 6.25a$

Ratio $= \frac{6.25}{4.18} = 1.50$ Choose 5 : 7.5 Taps $\frac{7.5}{5} = 1.50$

% mismatch (eq. 9.1 pg 286) = 0

---

c)



Both sets of CT's could be connected in delta, with the same CT ratios:

$I_{13.8} = \frac{1000}{13.8} = 72.4$

$= \frac{72.46}{30} = 2.42$ amps

$I_{2.4} = \frac{1000}{2.4} = 416.67a$

$= \frac{416.67}{200} = 2.08\,a.$

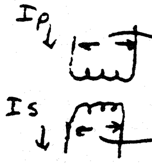
$\frac{2.42}{2.08} = 1.16$

Relay Taps should be changed to 5 : 5.5

$M = \frac{\frac{2.42}{2.08} - \frac{5.5}{5}}{1.1}\ 100$

$= 5.77\%$ ok.

d)



$I_p = \frac{1000}{13.8} = 72.46a.$

$I_s = \frac{1000}{2.4} = 416.67a.$

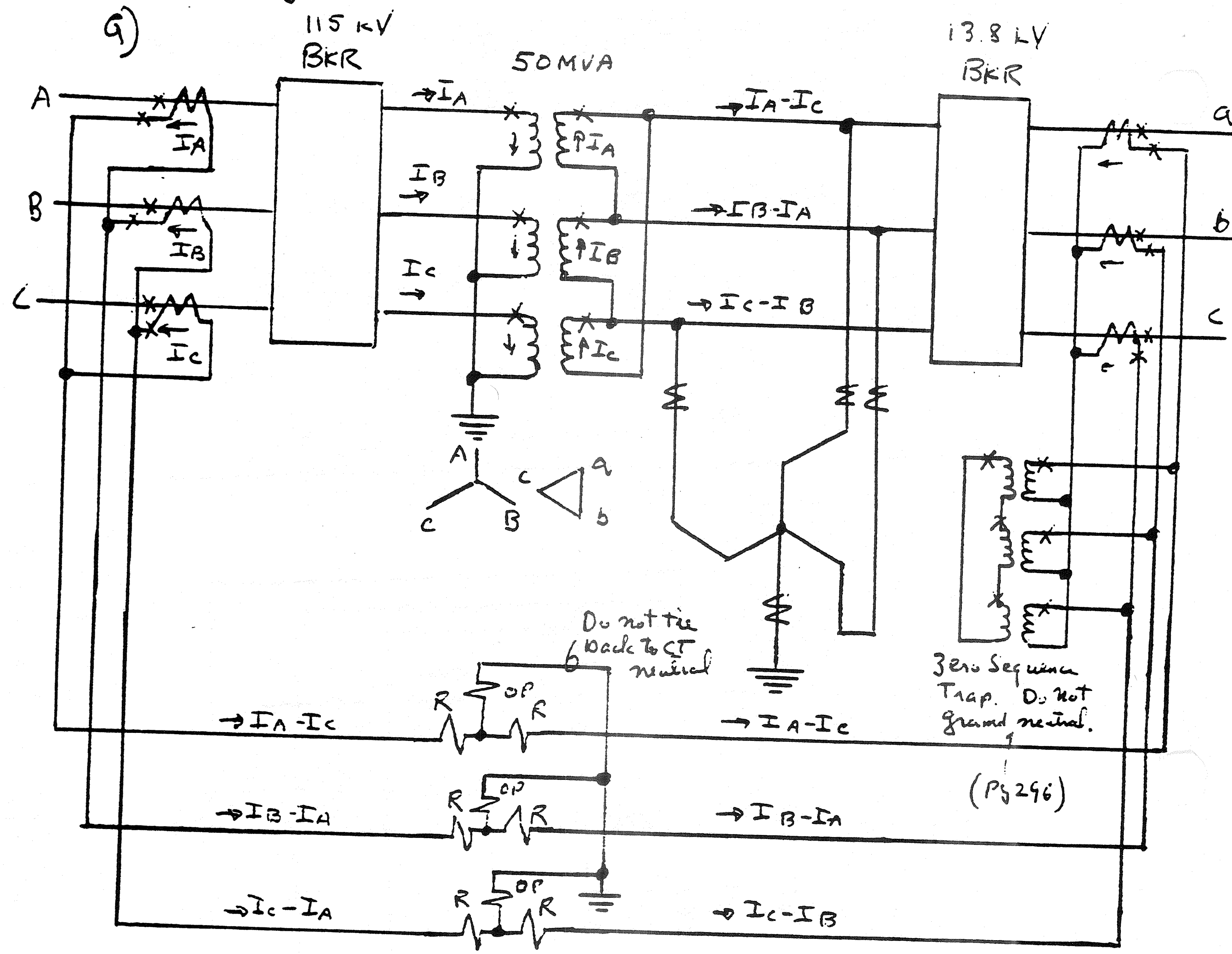
on a 3φ Base: KVA $= \sqrt{3}\ 72.46 \times 13.8 = 1732$ kVA

KVA $= \sqrt{3}\ 416.67 \times 2.4 = 1732$ kVA

Thus while transformers can supply 2000 KVA, The 3φ load can only receive 1732 or $\frac{3000}{\sqrt{3}}$ KVA.

Prob. 9.4 pg 518

a)



Alternate (1) 115 kV CT's in wye with Zero Seq. Trap, 13.8 kV CT's in delta

(2) Auxiliary wye-delta CT's could be used to provide phase shift.

Caution: always be sure zero seq (as well as pos. & neg.) of the power system have low impedance paths in the relay-secondary circuitry.

---

b) For faults on 13.8 kV system $X_1 = X_2 = .13$, $X_0 = .06\frac{50}{1.2} = 2.5$ pu on 50MVA.

$I_1 = I_2 = I_0 = \frac{1}{.13+.13+2.5} = \frac{1}{2.76} = .36$ pu

$I_a = 3I_0 = 1.09$ pu $= \frac{50,000}{\sqrt{3}\ 13.8} 1.09 = 2273.75$ amps at 13.8 kV

$I_{115} = \frac{50,000}{\sqrt{3} \times 115} = 251.02$ amps; choose 300,5 CT (60) $I_{sec} = \frac{251.02}{60} = 4.18$ amps.

$I_{relay} = \sqrt{3}\ 4.18 = 7.25$ amps.

$I_{13.8} = \frac{50,000}{\sqrt{3} \times 13.8} = 2091.85$ amps; choose 2200:5 CT (440) $I_{sec} = I_{relay} = \frac{2091}{440} = 4.75$.

Ratio $\frac{7.25}{4.75} = 1.53$; choose relay Taps ratio of 1.5 ; $M = \frac{1.53-1.5}{1.5} 100 = 1.75\%$ mismatch

Good.

Prob. 9.46 Cont'd. Current in differential relay for a 13.8kV gnd fault on the 115 kV side $I_1 = .36\angle 30^\circ$, $I_2 = .36\angle -30^\circ$ pu. (50MVA)

$I_A = .312 + j.180 + .312 - j.180 = .624$ pu.

$I_B = .36\angle 30+240^\circ\ (270) + .36\angle -30+120\ (90) = 0$

$I_C = .36\angle 30+120\ (150) + .36\angle -30+240\ (210) = -.312 + j.18 - .312 - j.18 = -j.624$ pu.

$I_A - I_C = .624 + .624 = 1.248$ pu $= \frac{50{,}000}{\sqrt{3} \times 115} 1.248 = 313.275$ amp at 115kV.

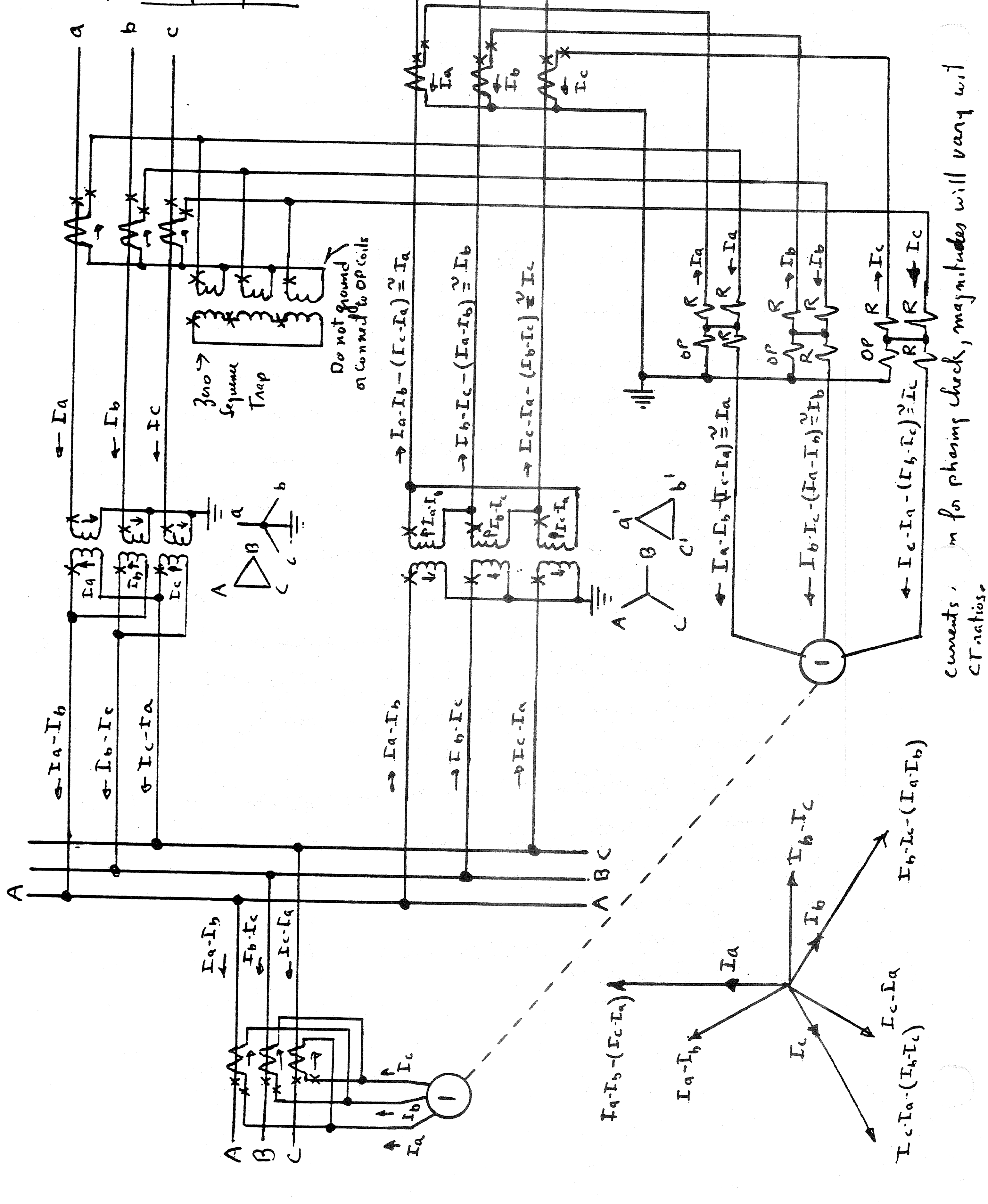
I in A diff. relay $= \frac{313.275}{60} = 5.22$ amperes. $\frac{5.22}{1.8} = 2.9$ times diff. relay Pick-up.

A time-overcurrent relay should be connected to the Zig-Zag neutral CT set to coordinate with the ground protection on the 13.8 kV feeder(s). It would also be desirable to use a ground differential protection (fig 9.10, pg 299) around the Zig Zag. Because of the smaller size 1.2 MVA Zig Zag relative to the 50 MVA Power Transformer, the CT's have a ratio based on 50 MVA load currents. Hence the Transf. differential is relative insensitive to Zig Zag bank faults. The gnd. diff. CT's could be 100:5, 150:5 or 200:5 to provide much more sensitive protection than the 2200:5 CTs used for the Bank diff.

The Zig-Zag neutral CT ratio and relay tap should be selected so that any normal zero sequence unbalance will not cause operation of the time-overcurrent (back-up) relay and any external ground faults on the feeder(s) will not result in exceeding the CT & relay thermal limits before the feeder protection can clear the faults.

Prob. 9.5 pg. 519   Many Variations are possible using either auxiliary CT's or zero sequence Trap.

a) One of Many Solutions:



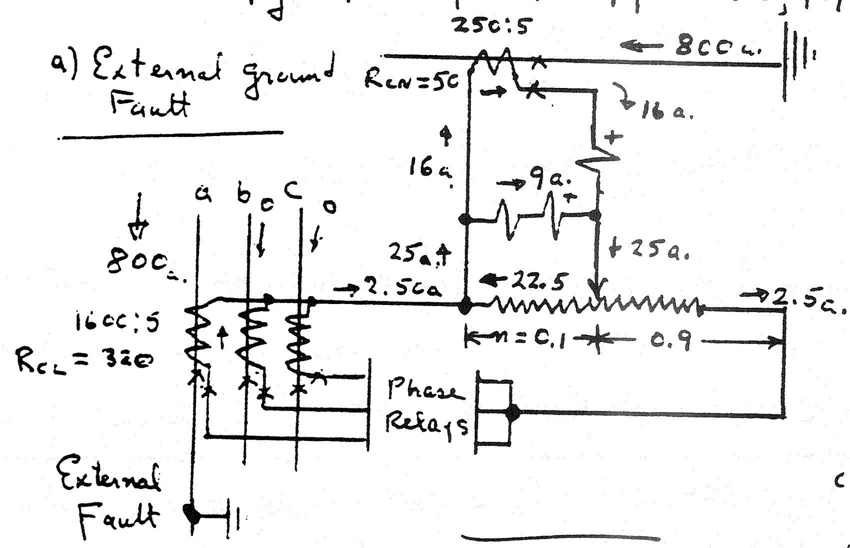
b) 1. The first concern if harmonic type relays are used is any possibility of a sympathetic magnetizing inrush? No problem as it is not possible to energize one bank, then later the second bank.

2. Best protection would be differential around each bank & a bus differential if CT's were available. However in the absence of individual high-side transformer breakers, any relay operation must trip out both banks & bus. Hence for the system as shown one differential is OK.

3. The system as shown was an actual one in service in a public type utility in the USA.

Problem 9.6 pg 519 figures 9-12 pgs 304-305, fig 9-13 pg 306.

a) External ground Fault



$\frac{R_{CN}}{R_{CL}} = \frac{50}{320} = .16$

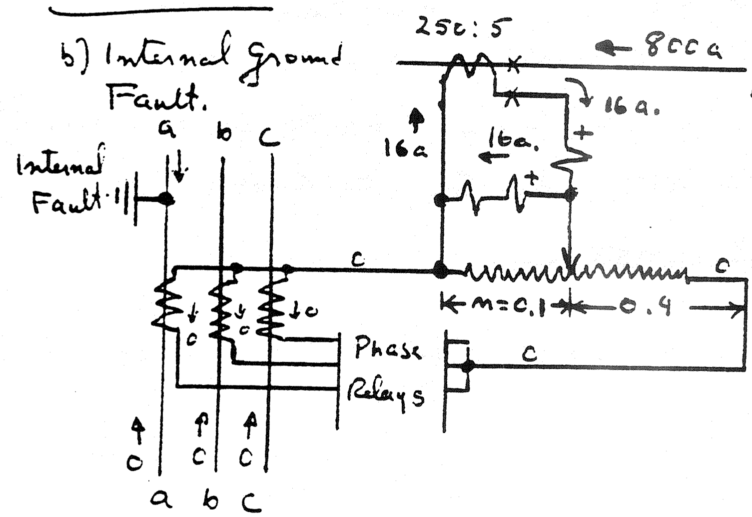
choose $n < .16$

Say 0.1

$\frac{.9}{.1} \times 2.5 = 22.50$ amps

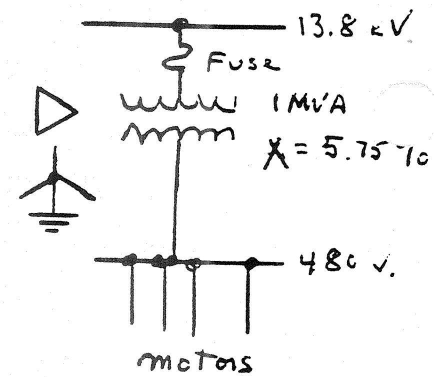
currents in directional non-operate directions.

b) Internal Ground Fault.



currents in directional operate directions.

Problem 9.7 pg. 520-521.



Source $X_1 = X_2 = .0355$ pu on 5MVA

Transformer $X = \frac{5}{1} x .0575 = .2875$ on 5MVA.

$\overline{.323 = X_1 = X_2}$

a) 480 volt bus fault: ground.

$$I_1 = I_2 = I_0 = \frac{1}{.323 + .323 + .2875}$$

$$= \frac{1}{.9335} = 1.071 \text{ pu.}$$

$$I_a = 3I_0 = 3.214 \text{ pu} = \frac{5000}{\sqrt{3}\ 0.48} = 19,324.5 \text{ amp: at 480 volts.}$$

For a solid fault (no arc) the current on the 13.8 kV side is

$$I_a = \frac{480}{13800\sqrt{3}} 19,324.5 = 388.13 \text{ amp. at 13.8 kV}$$ (reference Fig 9-19 pg 322)

b) with 150 volt arc, the fault current will be

$$I_a = 19,324.5\ \frac{277-150}{277} = 8860 \text{ amp. at 480 volts}$$

c) $$I_a = 8860 \times \frac{480}{\sqrt{3}\ 13,800} = 178 \text{ amperes at 13.8 kV}$$

d) Clearing time for 178 through the fuses is a little less than 175 seconds, almost 3 minutes. Hence considerable damage can occur from this 480 volt ground fault.

Problem 10.1 page 520-521: $I_F = 8000$ amps, $R_S = .296$ ohm, $R_L = .510$ ohm.
fig 10.9 pg 352, fig 5-10 pg 167 or use curve pg 18A above.
$n = 3$ circuits, $K = 0.7$ $N = 600:5 = 120$ $P = 2$

$$V_R = 1.6\,K(R_S + pR_L)\frac{I_F}{N} = 1.6 \times .7\,(.296 + 2 \times .510)\,\frac{8000}{120}$$

$= 98.26$ volts where $I_e = 0.1$ ampere

$$I_R = \frac{98.26}{1700} = .058 \text{ amps.}$$

$$I_{min} = (3 \times .1 + .058 + .02)120 = 45.36 \text{ amps.}$$ Min. Primary Operate Current

Problem 10.2 pg 521 increased to $n = 4$ (4 circuits)
fig 10.9 pg 352, fig 5.10 pg 167 or page 15A above.

$I_F = 10{,}000$ amps $R_S = .296$ ohm $R_L = .510$ ohm, $K = 0.7$, $P = 2$, $N = 120$ (600:5)

$$V_R = 1.6\,K\,(R_S + pR_L)\frac{I_F}{N} = 1.6 \times .7\,(.296 + 2 \times .510)\frac{10000}{120}$$

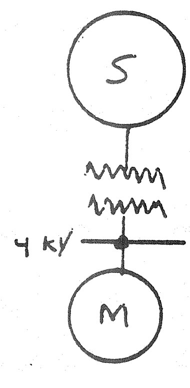
$= 122.83$ volts where $I_e = .115$ amp.

$$I_R = \frac{122.83}{1700} = .072 \text{ amps.}$$

$I_{min} = (4 \times .115 + .072 + .02)\,120 = 66.24$ amps min Primary operate current

---

Prob. 11.1 pg 521-522.



System — ON 100 MVA

431 MVA; $X_{1max} = X_{2max} = \frac{100}{431} = .232$ pu

113 MVA: $X_{1min} = X_{2min} = \frac{100}{113} = .885$ pu

2.5 MVA 13.8:4 kV 5.6% $X_T = \frac{100}{2.5} \times .056 = 2.24$ pu

2850 HP $I_{FL} = 362$ a. $I_{LR} = 1970$ a. $KVA = \sqrt{3} \times 362 \times 4 = 2508$ KVA.

$X_d'' = \frac{362}{1970} = .184$ pu on 2508 KVA

$= .184 \frac{100}{2.5} = 7.33$ pu on 100 MVA

Eq. 11.11 pg 366

$$X_{1S} = \frac{.866\,X_d''}{P_F P_R - 0.866}$$

$P_F = P_R = 2$ eq. 11.8 & 11.9 pg 365

$$X_{1S} = \frac{.866 \times 7.33}{4 - .866} = 2.03 \text{ pu or less}$$

actual $X_{1S} = .885 + 2.24 = 3.13$ pu. } Thus the instantaneous trip can not be
OR $X_{1S} = .232 + 2.24 = 2.47$ pu }
set as specified since X_{1S} of the connected system (3.13 or 2.47) > 2.03 calculated above.

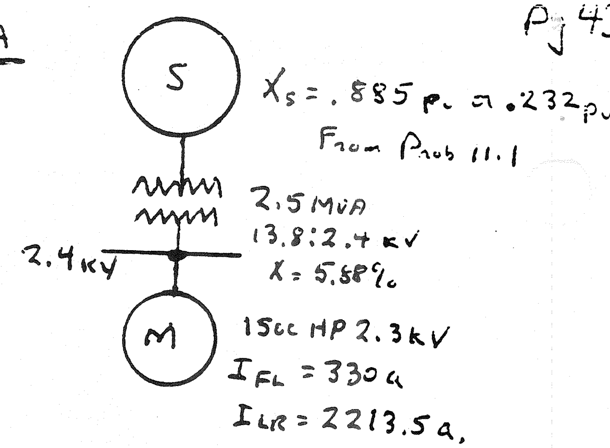
Prob. 11.2 pg 522. with a time delayed instantaneous trip use $P_R = 1.1$

from Prob. 11.1 $X_{1S} = \frac{.866 \times 7.33}{1.1 \times 2 - .866} = 4.76$ pu or less.

This type instantaneous trip can be applied since system X_{1S} values < 4.76 above.

Prob. 11.3 pg 522

ON 100 MVA



X_1 system $= .885 + \frac{100}{2.5} .0588$

$= .855 + 2.35 = 3.24$ pu

OR $= .232 + 2.35 = 2.58$ pu

KVA $= \sqrt{3} \times 2.3 \times 330 = 1315$ kVA.

$X_{1d}'' = \frac{330}{2213.5} = .149$ pu at 1,315 MVA

$= \frac{100}{1.315} \times .149 = 11.34$ pu on 100 MVA.

eq. 11.11 pg 366

$$X_{1S} = \frac{.866 X_d''}{P_F P_R - 0.866}$$

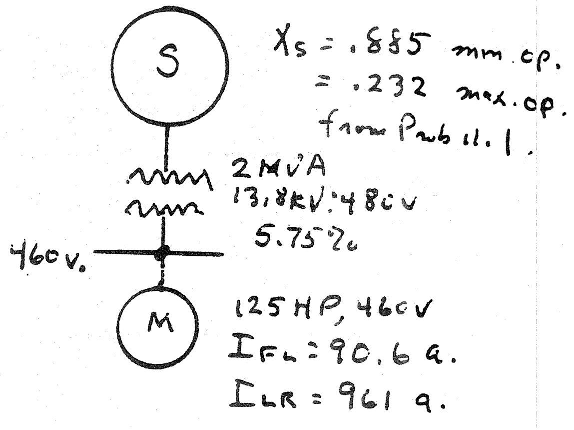
$P_F = P_R = 2$ (eqs. 11.8 & 11.9, pg 365)

$$X_{1S} = \frac{.866 \times 11.34}{4 - .0866} = 3.13 \text{ pu or less.}$$

Thus the instantaneous trip is marginal when the system is operating at its minimum condition but ok for maximum operating condition. Since minimum operation probably will be infrequent, application of the instantaneous trip is recommended.

Prob 11.4 pg 522

ON 100 MVA



2 MVA Transf. $X_T = \frac{100}{2} \times .0575$ pu

$= 2.875$ pu

X_1 system $= .855 + 2.875 = 3.76$ pu

OR $= .232 + 2.875 = 3.11$ pu

$$X_{1S} = \frac{.866 X_d''}{P_F P_R - 0.866}$$ eq. 11.11, pg 366

$P_F = P_R = 4$ eqs 11.8 & 11.9, pg 365

$$X_{1S} = \frac{.866 \times 130.6}{4 - .866} = 36.1 \text{ pu or less.}$$

KVA $= \sqrt{3} \times .46 \times 90.6$

$= 72.18$ KVA

$X_d'' = \frac{90.6}{961} = .094$ pu on 72 KVA

$= \frac{100}{.072} \times .094 = 130.6$ pu on 100 MVA

Thus the instantaneous trip unit is applicable since X_1 system of 3.76 or 3.11 < 36.1. Good application.

Prob. 11.5 pg 522 & 523. On 5 MVA Base.

from figure P11.5 Page 523:

138 kV system $X_{1S} = X_{2S} = \frac{5}{10,000} = .0005$ p.u.

10 MVA Transf. $X_T = \frac{5}{10} \times .07 = .0350$ pu

5 MVA Transf $X_T = .0550$ pu.

$X_1 = X_2$ To 4160 volt bus $= .0905$ pu.

Syn. motors: $KVA = 4 \times \sqrt{3} \times 250 = 1732.051$ KVA

$X_d'' = .24$ pu on 1732 KVA, 4kV $= \frac{5}{1.732}\left(\frac{4}{4.16}\right)^2 \times .24$

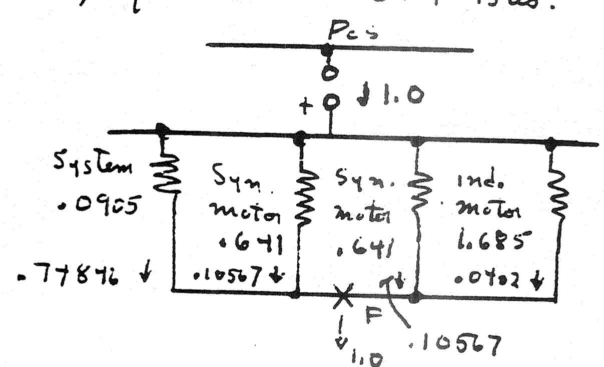
$= .641$ pu on 5MVA 4.16 kV.

induction motors: $KVA = \sqrt{3} \times 4.16 \times 70 = 504.373$ KVA

$X_d'' = \frac{5}{.5044} \times .17 = 1.685$ pu on 5MVA, 4.6KV.

neglect all 480 volt motor contributions as they are less than 50HP (per ANSI Standard C37.010 pg 34).

a) Solid 3ϕ fault on 4.16KV Bus! percent on 5MVA, 4.16kV.



$$\frac{.0905 \times .3205}{.411} = .07057 \text{ pu} \quad (.22019,\ .77981)$$

$$X_1 = \frac{.07057 \times 1.685}{1.75557} = .06774 \text{ pu} \quad (.04020,\ .95980)$$

$I_{3\phi} = \frac{1}{.06774} = 14.762$ pu $= \frac{5000}{\sqrt{3} \times 4.16} \times 14.762 = 10,244.03$ amps at 4.16KV.

b) The induction motor contributes 4.02% to the fault.
The synchronous motors, each contribute 10.57% to the fault.
74.85% is supplied by the system.
Practically, the induction motor can be ignored because of small contribution and fast decay of its internal voltage.

c) $X_1 = X_2 = .0905$ pu. from above. of source only

or $X_1 = X_2 = .06774$ pu for system including motors.

8 ohm Resistor in the 5 MVA Transformer neutral

$$R_{pu} = \frac{5 \times 8}{(4.16)^2} = 2.31 \text{ pu.} \; ; \; 3R = 6.93 \text{ pu.}$$

$$X_0 = 6.93 + .055 = 6.99 \text{ pu.}$$

$$I_1 = I_2 = I_0 = \frac{1}{.068 + .068 + 6.99} = \frac{1}{7.12} = .140 \text{ pu.}$$

$$= \frac{5000}{\sqrt{3} \times 4.16} \times .14 = 97.40 \text{ amps at } 4.16\text{KV.}$$

$I_a = 3I_0 = .421$ pu $= 292.2$ amperes at 4.16 kV.

compared to $I_{3\phi} = 14.76$ pu $= 10,244$ amperes at 4.16 kV from part a).

d) For The Induction motor: with $X_d'' = .17 = \frac{I_{FL}}{I_{LR}} = \frac{70}{I_{LR}}$

$I_{LR} = 411.76$ amperes at 4.16KV with $I_{FL} = 70$ choose 100:5 CTs

$I_{LR} = \frac{411.76}{20} = 20.59$ amps secondary.

Set phase instantaneous unit at $2 \times I_{LR} = 41.18$ a set 42 amps sec.

For the 3ϕ fault: Total fault current part a) $= 10,244$ amperes.

Thru the relays $I_{3\phi} = 10,244 \times .9598 = 9832$ amps Pri (less .0402 Motor Contribution)

$I_{3\phi} = \frac{9832}{20} = 491.6$ amps sec. This is $\frac{491.6}{42} = 11.7$ times pickup of the Phase relay.

For the ϕ gnd Fault. $\frac{292.2}{20} = 14.6$ amp. Sec. use 0.5 or 1.0 amp Setting.

For the Synchronous Motors. with 250 amp Full load use 300:5 (60) CTs

Set phase instantaneous at $2 I_{LR} = 3000$ a pri $= 50$ amps Sec.

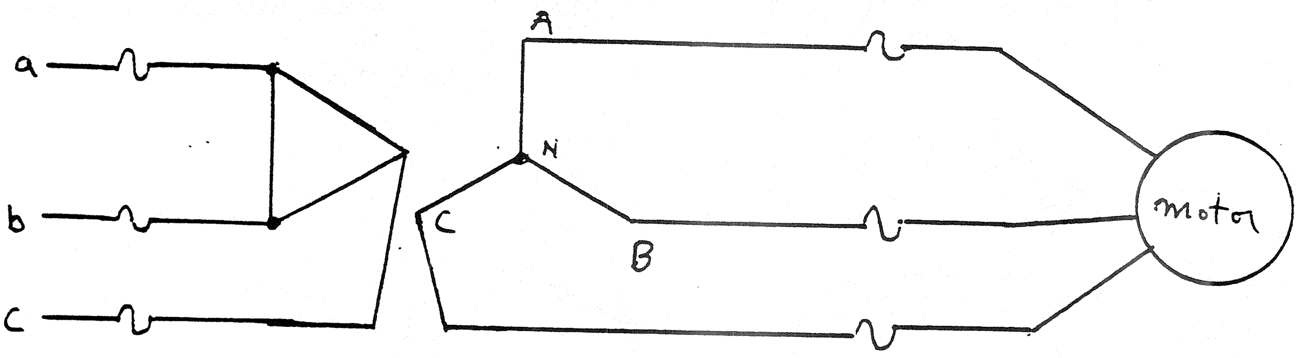
Thru the relays $I_{3\phi} = 10,244 \times .89433 = 9161.5$ amps (less .10567 motor Contribution)

$= \frac{9161.5}{60} = 153$ amps sec. This is on 3X relay pick up.

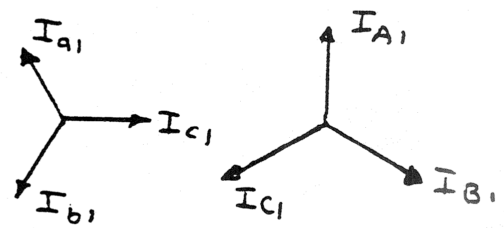
differential relays recommended for the Synchronous motors.

$I_{\phi gnd} = \frac{292.2}{60} = 4.87$ amp Sec. use 0.5 or 1.0 amp setting.

Problem 11.6 pg 522-523. Connections shown on page 523.

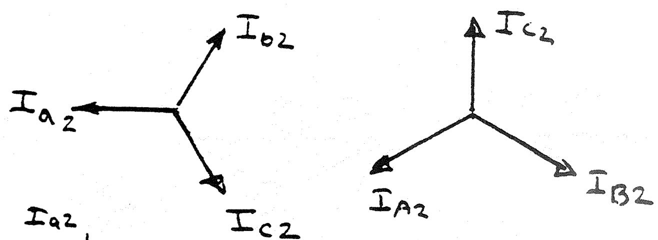


Pos. Sequence

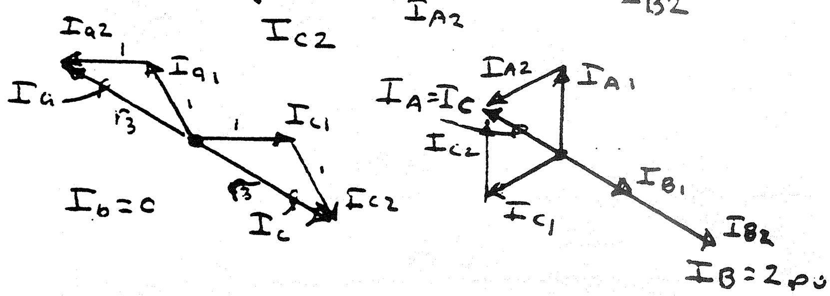


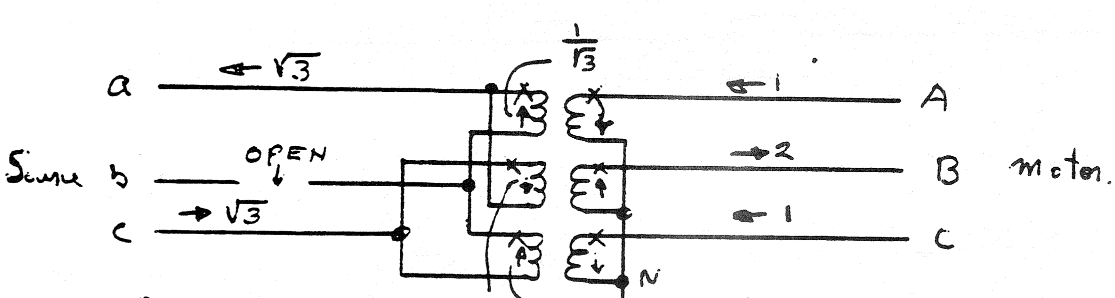
I_{A1} lags I_{a1} 30° as shown

a) when b fuse opens, negative sequence results so that $I_{b1} + I_{b2} = I_b$ and $I_b = 0$ hence $I_{b2} = -I_{b1}$



Therefore I_{A2} leads I_{a1} 30°





$\frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}$ $\quad \frac{2}{\sqrt{3}} \quad 1/\sqrt{3}$

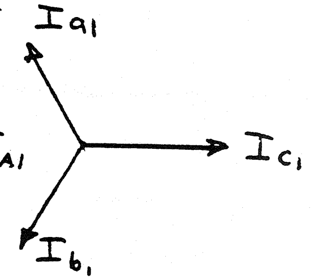
checks phasors above

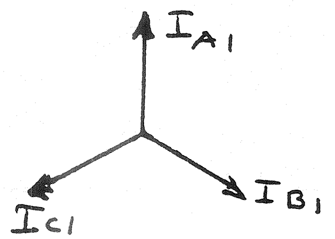
Problem 11.6 cont'd

b) Phase A fuse open. Negative sequence results so that

$$I_A = I_{A1} + I_{A2} = 0, \quad I_{A2} = -I_{A1}.$$

I_{a1} leads I_{A1} by 30°

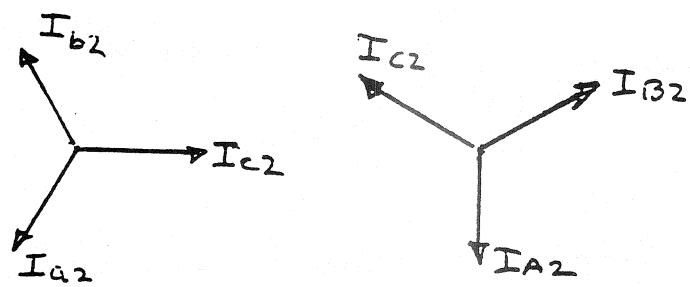


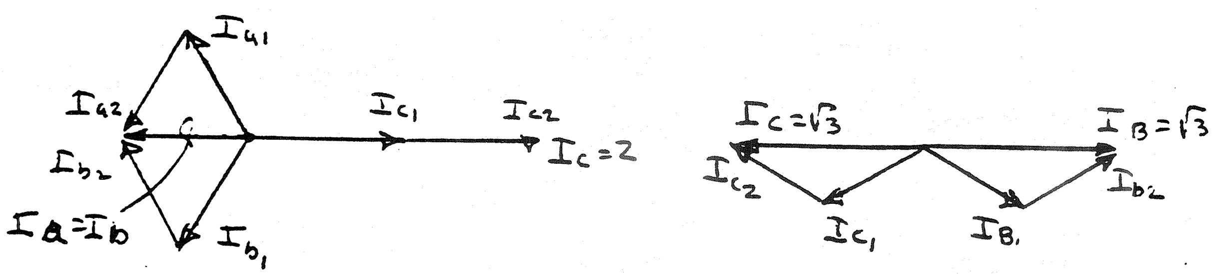


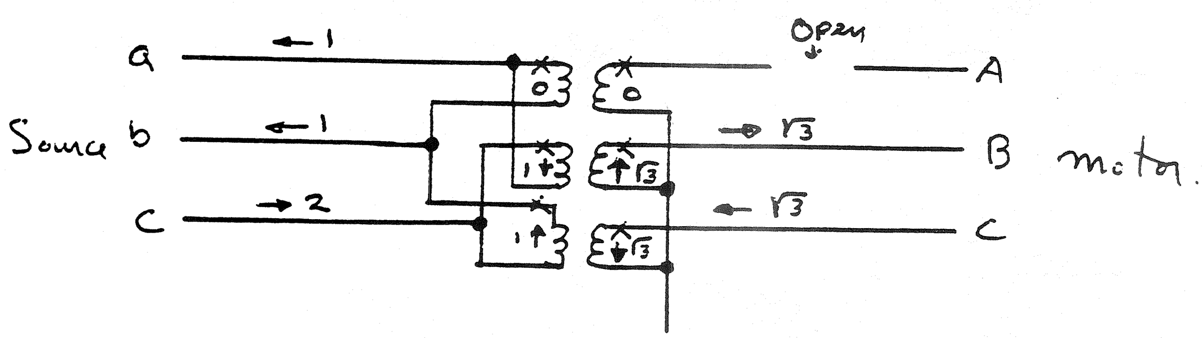
I_{A1} lags I_{a1} 30°

Therefore

I_{a2} lags I_{A2} by 30°







checks phasors above.

c) Grounding neutral does not affect the analysis. No zero sequence current flows unless the motor neutral is grounded which is not normal.

Problem 12.1 Page 524-526.

Since the coordination curves will be plotted in 12.5 kV amperes, the characteristics of the 46 kV Fuse of Table 3 pg 525 should be translated into equivalent 12.5 kV amperes for various faults: From Fig 9-19 page 322;

For 3ϕ faults: $I_{12.5kV} = \frac{46}{12.5} I_{46kV} = 3.68 I_{46kV}$.

For $\phi\phi$ faults $I_{12.5kV} = \frac{46}{12.5} \times .866 I_{46kV} = 3.19 I_{46kV}$

For ϕgnd faults $I_{12.5kV} = \frac{46}{12.5} \sqrt{3} I_{46kV} = 6.37 I_{46kV}$.

Thus for the 46 kV high side fuse, the times for 12.5 kV amperes are (Table 3)

Time in Seconds.	3ϕ Fault amperes at 12.5 kV	$\phi\phi$ Fault amperes at 12 kV	ϕGnd Fault amperes at 12 kV
300	130 x 3.68 = 478.40	130 x 3.19 = 414.31	130 x 6.37 = 828.61
10	260 x 3.68 = 956.80	260 x 3.19 = 828.61	260 x 6.37 = 1657.23
1	500 x 3.68 = 1840.00	500 x 3.19 = 1593.49	500 x 6.37 = 3186.97
0.1	1500 x 3.68 = 5520.00	1500 x 3.19 = 4780.46	1500 x 6.37 = 9560.92

Transformer Bank rated current $I = \frac{3750}{\sqrt{3}\, 12.5} = 173.2$ amps at 12.5 kV from Table 9.2 page 316, this bank is Category II, frequent fault, Fig 9.16 a & b, page 314-315. This limit curve is plotted along with the 46 kV fuse, 30 ampere fuse* and circuit recloser characteristics on log-log fuse paper (Pg 50). Fault 1 and Fault 2 phase & ground values are also plotted. This provides graphic data for relay setting & coordination.

Phase relays

With transformer rated current of 173 amperes choose 200:5 CTs unless there are forced air, forced oil overload possibilities not specified in the problem.

$I_S = \frac{173.2}{40} = 4.33$ amps: Select time overcurrent relay, tap 6, $\frac{6}{4.33} = 1.39$

minimum operate current is $40 \times 6 = 240$ amps at 12.5 kV.

Set the phase time overcurrent relays to provide a minimum of 0.2 sec. coordination with the circuit recloser. The circuit recloser operates in 0.345 sec for the 1300 amp 3ϕ fault. Therefore the time overcurrent phase relay should not operate in less than $0.345 + 0.2 = 0.545$ sec. at 1300 amps, or at $\frac{1300}{240} = 5.42$ multiple.

* Table 2 page 525 1000 amp time is 0.06 sec. not .6 sec.

From relay curve fig. P12.11 page 531 Time dial 2.5 provides a time of approx .55 sec at 5.42 multiple.

For the max. fault: $\frac{2300}{240} = 9.58$ mult. at TD 2.5 time is .32 sec. from curve. Plot the 2.5 Time dial curve. This coordinates well between the recloser which can only operate for Fault 2 or lesser currents, and the primary fuse.

The phase instantaneous trip unit can be set to not operate on fault 2 to avoid opening the main breaker for faults beyond the recloser, OR with the 30 amp Load tap located at or near the recloser, the instantaneous unit can be set to operate on faults 2 and lock out for fuse saving. For this solution the first option was selected so set IT at 1.15 × 1300 = 1500 amps at 12.5 kV.

Ground Relays

Must be set above any operating unbalance. When the circuit recloser is open the maximum single phase load is 30 amp or less.

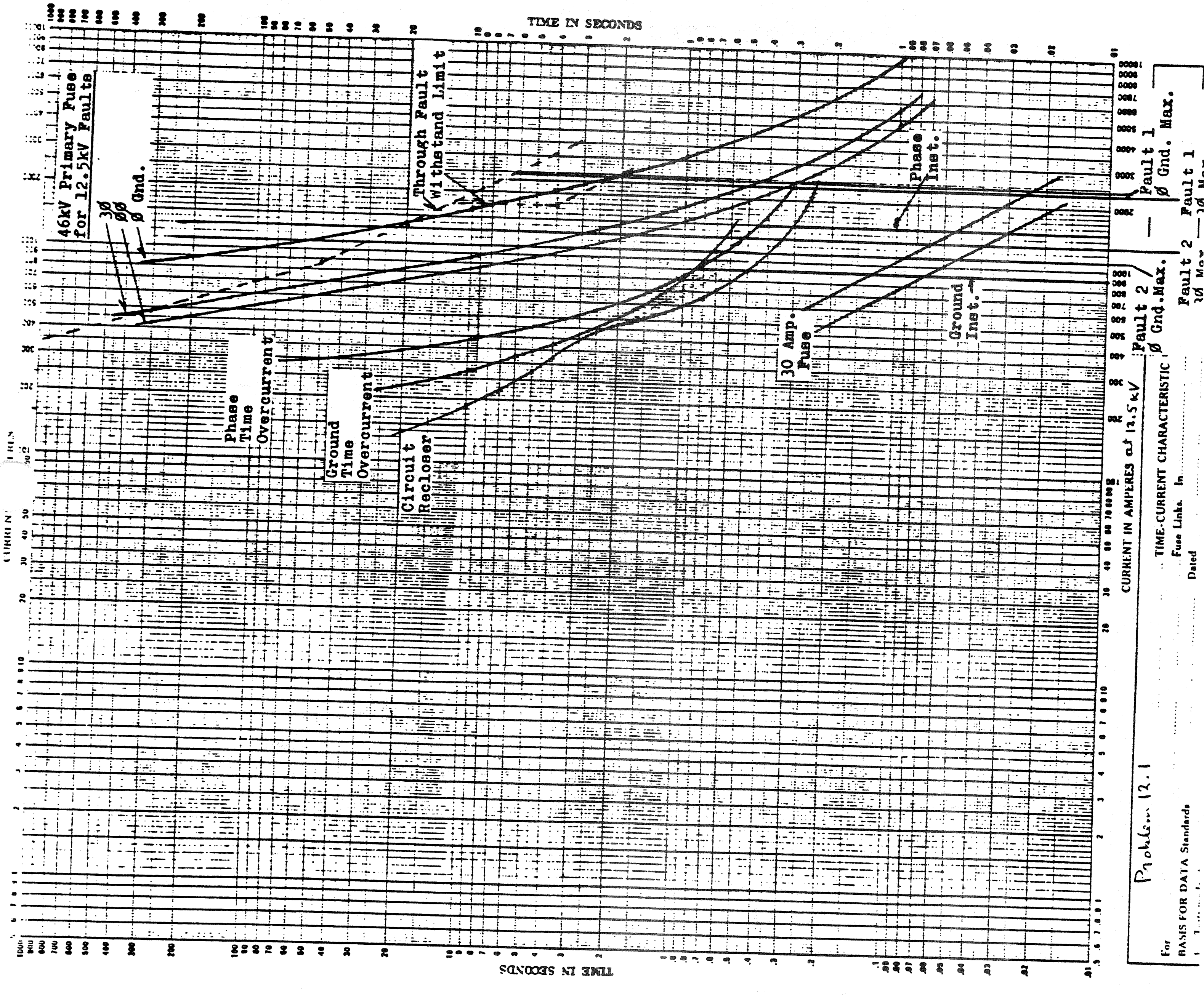
$\frac{30}{40} = .75$ Hence select Tap 1.0.

minimum gnd relay operate current is 1 × 40 = 40 amps at 12.5 kV.

This setting will not provide coordination, so drawing in a ground time overcurrent curve as shown that will coordinate with the circuit recloser, this requires Tap 4 and Time dial 4 of fig P12.11. With the 200:5 CTs the primary pick up is 40 × 4 = 160 amps at 12.5 kV.

Select ground instantaneous trip at approx 80% of Fault 2 to protect branch fuse during the most probable ground transient faults. It should be locked out after the first operation. Thus permanent or reoccurring ground faults beyond the fuse or recloser will be cleared by the fuse or recloser backed up by the overcurrent time units.

1110 × .8 = 888 amperes: set 900 amps = $\frac{900}{40}$ = 22.5 a. Sec.

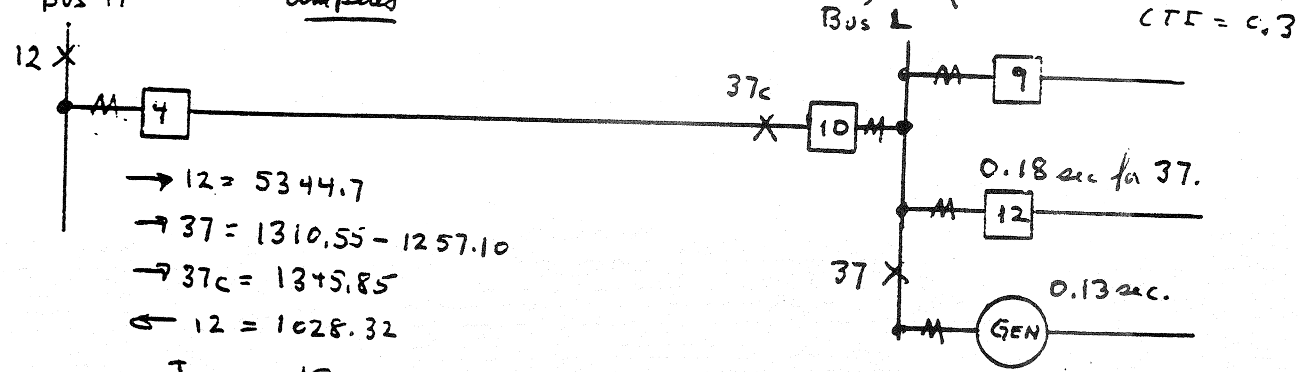

TIME IN SECONDS
46kV Primary Fuse for 12.5kV Faults
3Ø
ØØ
Ø Gnd.
Through Fault Withstand Limit
Phase Inst.
Fault 1 Ø Gnd. Max.
Fault 1 3Ø Max
Fault 2 Ø Gnd.Max.
Fault 2 3Ø Max
Ground Inst.
30 Amp. Fuse
Phase Time Overcurrent
Ground Time Overcurrent
Circuit Recloser
CURRENT IN AMPERES at 12.5kV
TIME-CURRENT CHARACTERISTIC
Problem 12.1
BASIS FOR DATA Standards
TIME IN SECONDS

Problem 12.2 pg. 526

Based on fig 12.4 page 403
Coordinate breakers 4, 6 & 9.

Currents in Primary amperes

Coord. Time Int. CTI = 0.3



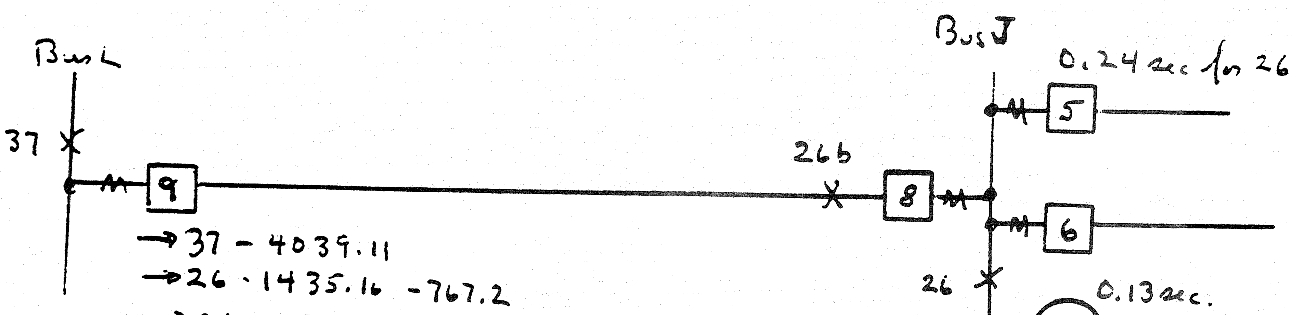
→ 12 = 5344.7
→ 37 = 1310.55 – 1257.10
→ 37c = 1345.85
← 12 = 1028.32
I_{STM} = 150

With load of 150 a. at breaker 4 select 200:5 (40) CT's; $\frac{150}{40} = 3.75a$.
Choose Tap 5. Primary pick up 5 × 40 = 200 amps.

Close-in fault (12) $\frac{5344.7}{200} = 26.72$ mult.

For bus fault (37) max $\frac{1310.55}{200} = 6.55$ mult min $\frac{1257.10}{200} = 6.29$ mult.

Operating time for breaker 9 is not known yet so assume it will not exceed 0.18 sec. So time for 4 for the far bus fault should not be less than 0.18 + 0.3 = 0.48 sec. From fig. 12.6 pg 407; mult 6.55 & 0.48 requires Time Dial 2, where time for 6.55 mult. is 0.62 sec. For the close-in fault (12) of mult. 26.72 est. time at 0.3 sec. for breaker 4.



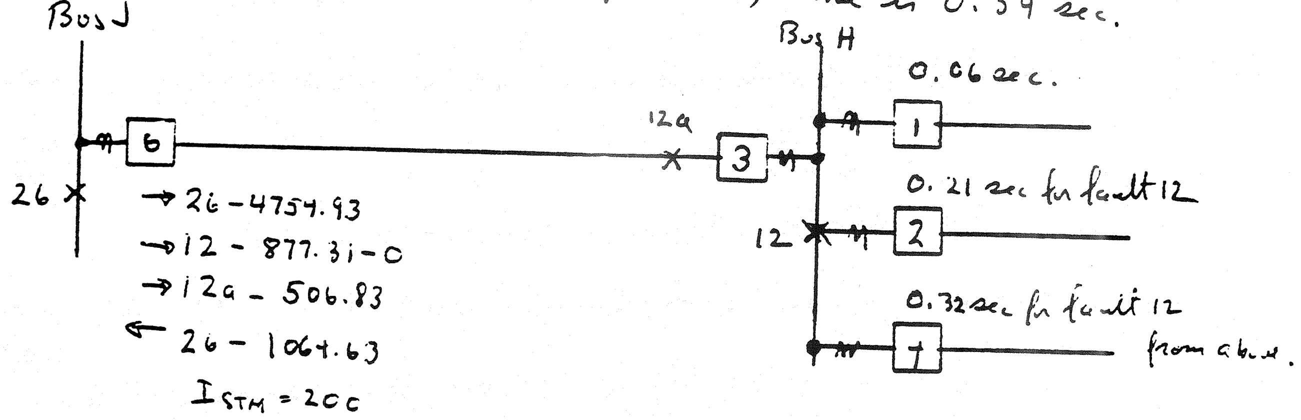
→ 37 – 4039.11
→ 26 – 1435.16 – 767.2
→ 26b – 854.67
← 37 – 1481.8
I_{STM} = 150

With load of 150 amps at breaker 9, select 200:5 (40) CT's; $\frac{150}{40} = 3.75a$
Choose Tap 5 Primary pick-up of 5 × 40 = 200 amps.

Close-in fault (37) $\frac{4039.11}{200} = 20.20$ mult.

For bus fault (26) max $\frac{1435.16}{200} = 7.18$, min $\frac{767.2}{200} = 3.84$ mult.

operating time for breaker 6 is not known so assume it will be not more than 0.24. So time for 9 for the far bus fault should not be less than 0.24 + 0.3 = 0.54 sec. From fig 12.6; mult. 7.18 & .54 sec requires Time Dial 2 where time for 7.18 mult is 0.58 sec. For the close-in fault (37) time is 0.34 sec.



with load of 200 amps at breaker 6, select 250:5(50) CTs, $\frac{200}{50} = 5$ a. Choose Tap 6. Primary pickup = 6 × 50 = 300 amps.

close-in fault (26) $\frac{4754.93}{300} = 15.85$ mult.

For bus fault (12) $\frac{877.31}{300} = 2.92$ mult. relay will not see the far bus min fault until breaker 3 opens, Then $\frac{506.83}{300} = 1.69$ mult.

Time for breaker 6 to operate on far bus fault should not be less than 0.3 + 0.32 = 0.62 sec. From fig 12.6 mult 2.92 & Time 0.62 sec requires Time Dial 1 where time for 2.92 mult is 0.8 sec. For the close in fault (26) mult 15.85 Time Dial 1 time is .2 sec.

Breaker 9 relay times for a close-in fault is 0.34 sec. Hence Breaker 4 relays for the far-bus fault should be 0.34 + 0.3 = 0.64 sec Breaker 4 relay time for the far bus fault is 0.62 sec. This small difference does not justify changing the settings.

Problem 12.3 pg 526   fig 12.4 pg 403

Set instantaneous trip. for Breakers 4, 6 & 9.

use $k = 1.2$

Breaker 4. Set instantaneous unit at $1.2 \times 1310.55 = 1573$ amps

$= \frac{1573}{40} = 39$ amp. sec.

non directional type can be used since $1573 > 1028.32$ the near bus fault max.

Breaker 9 set instantaneous unit at $1.2 \times 1435.16 = 1722$ amp.

$= \frac{1722}{40} = 43$ amps sec.

non directional type can be used since $1722 > 1481.8$ the near bus max fault.

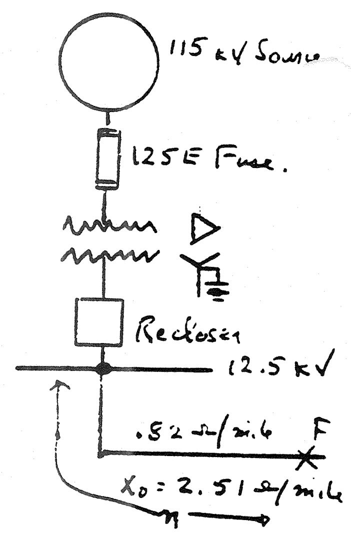
Breaker 6 set instantaneous unit at $1.2 \times 877.31 = 1052$ amps.

$\frac{1052}{50} = 21$ amp sec.

directional type required since 1064.63 a: near bus max. is > 1052 amp

---

Problem 12.4 pg 526.   100 MVA Base.



To 12.5kV Bus: $X_1 = X_2 = 0.63$ pu
$X_0 = 0.6$ pu.

Z_1 line $= \frac{MVA\, Z_\Omega}{kV^2} = \frac{100 \times .82}{12.5^2} = .525$ pu/mile

Z_0 line $= \frac{100 \times 2.51}{12.5^2} = 1.606$ pu/mile

$X_1 = X_2 = .63 + .525n$
$X_0 = .6 + 1.606n.$

300 amps primary on 115 kV side $= \frac{300 \times 115}{12.5}\sqrt{3}$ (fig 9-19c pg 322)

$= 4780.46$ amperes for a 12.5 kV ground fault.

$I_{Base} = \frac{100,000}{\sqrt{3}\, 12.5} = 4618.80$ amps at 12.5 kV

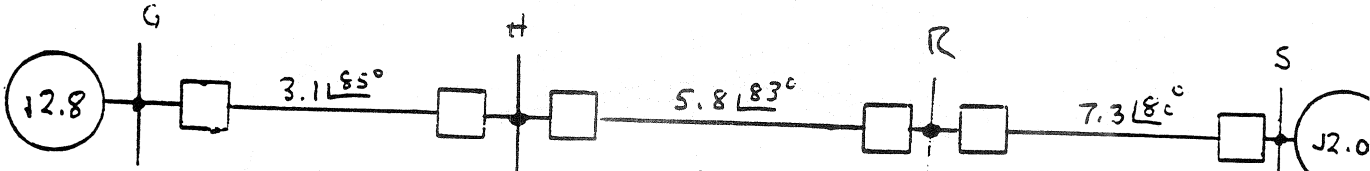
$I_{\phi G} = \frac{4780.46}{4618.8} = 1.035$ per unit.

$I_1 = I_2 = I_0 = \frac{1.035}{3} = \frac{1}{2(.63+.525n)+.6+1.606n} = .345$ pu. $= \frac{1}{1.86+2.656n}$

$.6417 + .91632n = 1$ $n = \frac{1-.6417}{.91632} = .39$ miles

no practical line protection will be available by the high side fuse.

Problem 12.5 pg 526-527



Values in secondary ohms.

a) <u>Station H</u> looking into line HR, $Z_{HR} = 5.8\angle 83° = .707 + j5.757$ ohms.

<u>Zone 1</u>: $90\% Z_{HR} = 5.22\angle 83° = .636 + j5.181$ ohms.

<u>Zone 2</u>: $Z_{HR} + \frac{1}{2} Z_{RS} = .707 + j5.757 + \frac{1}{2}(1.268 + j7.189)$

$= 1.341 + j9.351 = 9.45\angle 81.84°$ ohms.

<u>Zone 3</u>: $Z_{HR} + 1.2 Z_{RS} = .707 + j5.757 + 1.522 + j8.627$

$= 2.229 + j14.384 = 14.56\angle 81.19°$ ohms

Station R looking into line RH,

<u>Zone 1</u>: 90% of $Z_{RH} = 5.22\angle 83° = .636 + j5.181$ ohms (same as for H.)

<u>Zone 2</u>: $Z_{RH} + \frac{1}{2} Z_{HG} = .707 + j5.757 + \frac{1}{2}(.270 + j3.088)$

$= .842 + j7.301 = 7.35\angle 83.42°$ ohms sec.

<u>Zone 3</u>: $Z_{RH} + 1.2 Z_{HG} = .707 + j5.757 + 1.2(.270 + j3.088)$

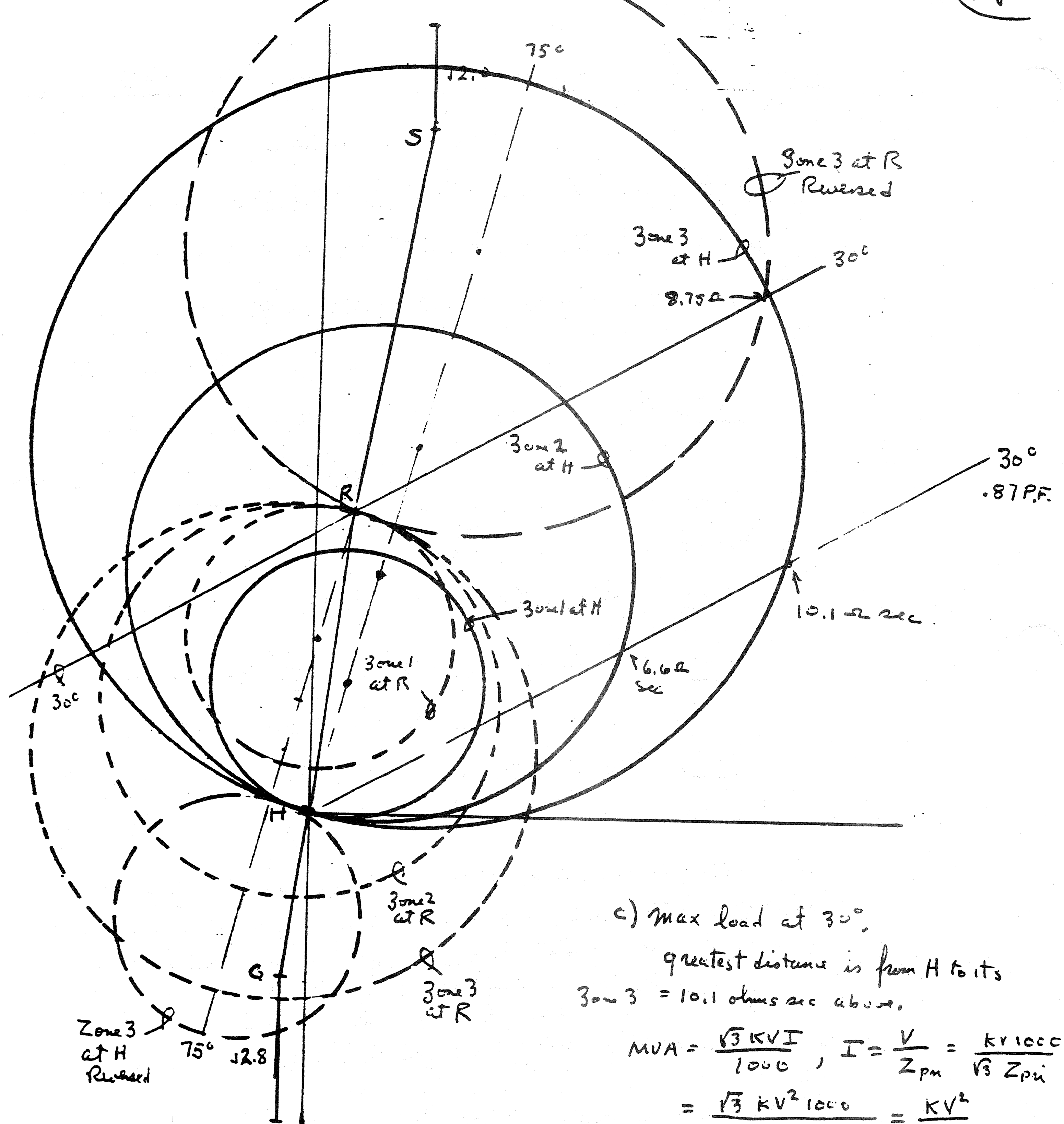
$= 1.031 + j9.463 = 9.52\angle 83.78°$ ohms.

b) As indicated the calibrated relay settings are at 75° while the line HR is 83°. While the relay angle can be changed this is not recommended. From formula 6.3 pg 197 if the relay is set for 5.22 ohms with angle at 75°, the reach at 83° will be $5.22(\cos 83°-75°) = 5.22 \cos 8° = 5.17$ ohms, covering 89% of the line instead of 90%. OR the relay can be set at $\frac{5.22}{\cos 8°} = 5.27$ ohms at 75° to give reach of 90% at 83°.

Equivalent Z_S settings for 75°:

	<u>at H</u>	<u>at R</u>
Zone 1	$\frac{5.22}{\cos 8°} = 5.28\angle 75°$, $Z_1 = 2.64 + 2.64\angle\phi$	same as for H.
Zone 2	$\frac{9.45}{\cos 6.84°} = 9.52\angle 75°$, $Z_2 = 4.76 + 4.76\angle\phi$	$\frac{7.35}{\cos 8.42°} = 7.43\angle 75°$, $Z_2 = 3.72 + 3.72\angle\phi$
Zone 3	$\frac{14.56}{\cos 6.19°} = 14.65\angle 75°$ $Z_3 = 7.32 + 7.32\angle\phi$	$\frac{9.52}{\cos 8.78°} = 9.63\angle 75°$, $Z_3 = 4.82 + 4.82\angle\phi$

Prob 12.5 b Cont'd.



c) max load at 30°,
greatest distance is from H to its Zone 3 = 10.1 ohms sec above.

$$MVA = \frac{\sqrt{3}\, KV\, I}{1000}, \quad I = \frac{V}{Z_{pri}} = \frac{KV\, 1000}{\sqrt{3}\, Z_{pri}}$$

$$= \frac{\sqrt{3}\, KV^2\, 1000}{\sqrt{3}\, 1000\, Z_{pri}} = \frac{KV^2}{Z_{pri}}$$

(note $R_c = 80$ Isec of 5 amps is 400 amps Pri. Then $MVA = \frac{\sqrt{3} \times 115 \times 400}{100} = 79.67$)

$Z_{pri} = Z_{sec} \frac{R_v}{R_c}$ so $MVA = \frac{KV^2}{Z_{sec}} \frac{R_c}{R_v}$

with $R_v = 1000$, $KV = 115$; $MVA = \frac{115^2 \times 80}{10.1 \times 1000} = \underline{104.75\ MVA}$

Problem 12.6 page 527.

a) Zone 1 and 2 settings at H and R are as in Problem 12.5 above.

at H: Zone 3 reversed $1.5 \times Z_{HG} = 1.5 \times 3.1\angle 85^\circ = 4.65\angle 85^\circ \cong 4.72\angle 75^\circ$
$= 2.36 + 2.36\angle\emptyset$ ohms.

at R: Zone 3 reversed $1.5 \times Z_{RS} = 1.5 \times 7.3\angle 80^\circ = 10.95\angle 80^\circ \cong 10.99\angle 75^\circ$
$= 5.5 + 5.5\angle\emptyset$ ohms.

b) These are plotted on Page 55 chart.

c) Now for line HR, Zone 2 is the limit for load = 6.6 Ω sec.

$$MVA = \frac{KV^2 R_c}{Z_{sec} R_V} = \frac{115^2 \times 80}{6.6 \times 1000} = \underline{160.30\ MVA}.$$

(note with $R_c = 80$ 5 amps secondary current = 400 amps pri.
$MVA = \frac{\sqrt{3} \times 115 \times 400}{1000} = 79.67$ MVA)

Problem 12.7 pgs 527-528. at Station M

				Z_{actual}
a) Fault 1 max.	$Z_{apparent} =$	$\frac{55 \times 80 + 30 \times 260}{80}$	$= 152.50\%$	85%
Fault 1 min	$Z_{app} =$	$\frac{55 \times 50 + 30 \times 195}{50}$	$= 172.00\%$	85%
Fault 3 max	$Z_{app} =$	$\frac{55 \times 43 + 40 \times 210}{43}$	$= 250.35\%$	95%
Fault 3 min	$Z_{app} =$	$\frac{55 \times 25 + 40 \times 160}{25}$	$= 311.00\%$	95%

The reach of relay M into the remote lines is

$$\%\ reach = \frac{Z_{setting} - Z_{MS}}{Z_{Total} - Z_{MS}} 100$$

Z_{Total} can be Z_{actual} (no infeed) or $Z_{apparent}$.

b) % reach of Relay M for line SL:

with infeed as shown. — no infeed

max. Oper. Zone 2: % reach $= \frac{70-55}{152.5-55}100 = 15.3\%$ — % Reach $= \frac{70-55}{85-55}100 = 50\%$

min Oper. Zone 2: % reach $= \frac{70-55}{172-55}100 = 12.82\%$ — % Reach $= \frac{70-55}{85-55}100 = 50\%$

max oper. Zone 3: % reach $= \frac{100-55}{152.5-55}100 = 46.15\%$ — % Reach $= \frac{100-55}{85-55}100 = 150\%$

min. Oper Zone 3: % reach $= \frac{100-55}{172-55}100 = 38.46\%$ — % Reach $= \frac{100-55}{85-55}100 = 150\%$

---

b) % reach of Relay M for line SP

with infeed as shown — no infeed

max. Oper. Zone 2: % reach $= \frac{70-55}{250.35-55}100 = 7.68\%$ — % Reach $= \frac{70-55}{95-55}100 = 37.5\%$

min. Op. Zone 2: % reach $= \frac{70-55}{311-55}100 = 5.86\%$ — % Reach $= \frac{70-55}{95-55}100 = 37.50\%$

max Op. Zone 3: % reach $= \frac{100-55}{250.35-55}100 = 23.04\%$ — % Reach $= \frac{100-55}{95-55}100 = 112.5\%$

min. Op Zone 3: % Reach $= \frac{100-55}{311-55}100 = 17.58\%$ — % Reach $= \frac{100-55}{95-55}100 = 112.5\%$

c) Zone 3 set 100% Z; ohms $= \frac{KV^2\,\%Z}{100\,MVA} = \frac{161^2 \times 100}{100 \times 100} = 259.21$ ohms Primary

with relay set at 75°, reach at 30° for max load.

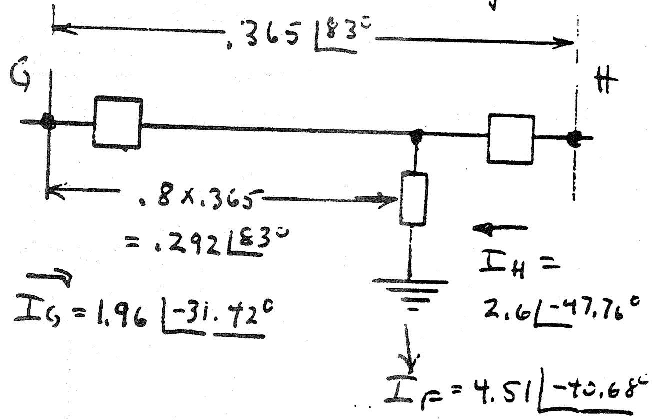
$$Z_{30} = 259.21\,(\cos 75^\circ - 30^\circ) = 183.29 \text{ ohms primary}$$

$$MVA = \frac{KV^2}{Z_{pri}} = \frac{161^2}{183.29} = 141.42 \text{ MVA.}$$

Normal 5 amp sec. = 5 × 100 = 500 amps Pri.

$$MVA = \frac{\sqrt{3} \times 161 \times 500}{1000} = 139.43 \text{ MVA.}$$

Problem 12.8 pages 528-529



R_{arc} = 12 ohms

$= \frac{100}{115^2} 12 = .09074$ pu.

values in per unit 100MVA, 115 kV.

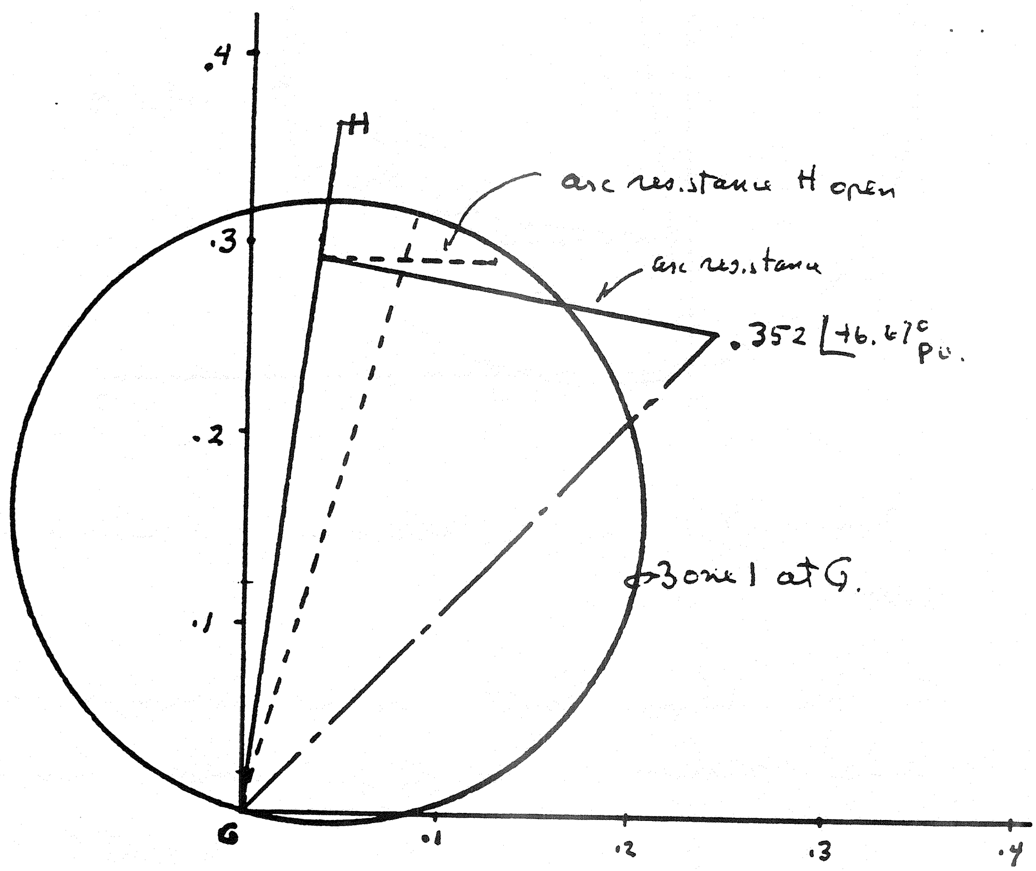
a) $Z_{1G\ apparent} = \frac{I_G Z_{GH} \times .8 + I_F R_{arc}}{I_G}$

$= .292\angle 83° + \frac{4.51\angle -40.68°}{1.96\angle -31.42°} \times .09074 = .03559 + j.28982 + 2.30102 \times .09074 \angle -9.26°$

$= .03559 + j.28982 + .20879\angle -9.26° = .03559 + j.28982 + .20607 - j.03360$

$= .24165 + j.25622 = .352\angle 46.67°$ pu.

b)

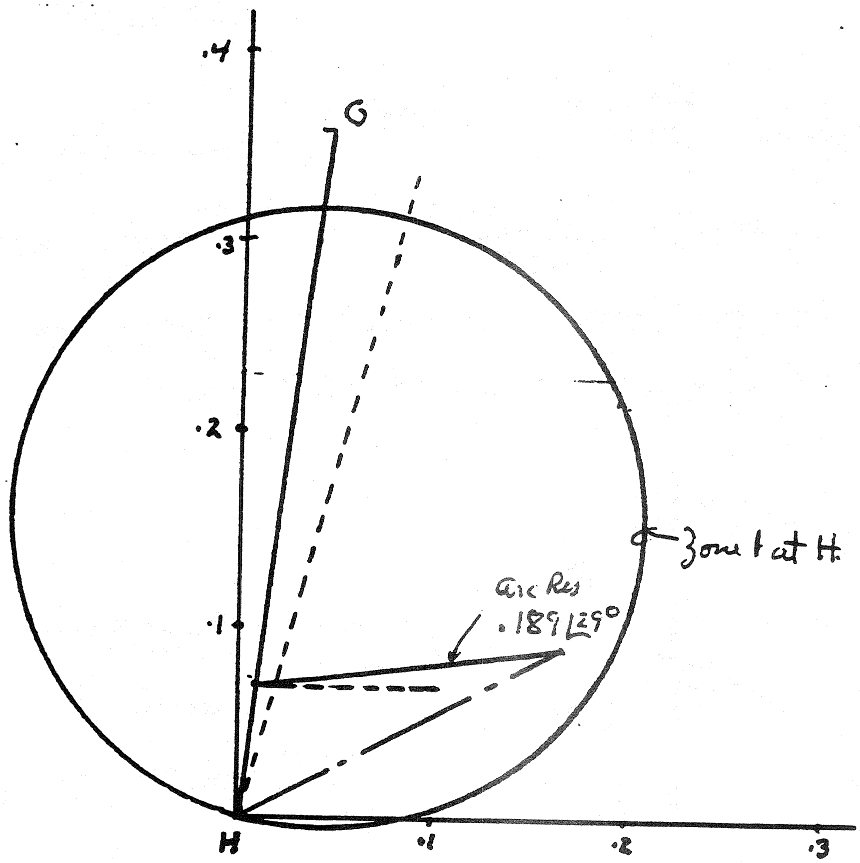


Zone 1 at G cannot operate as Z_{1G} apparent is outside the operating circle.

c) $Z_{1H} = \dfrac{I_H \times .2 Z_{GH} + I_F R_{arc}}{I_H} = .2 \times .365\angle 83^\circ + \dfrac{4.51\angle -40.68}{2.6\angle -47.76} \times .0..$

$= .073\angle 83^\circ + .15740\angle 7.08^\circ = .00890 + j.07246 + .15620 + j.01940$

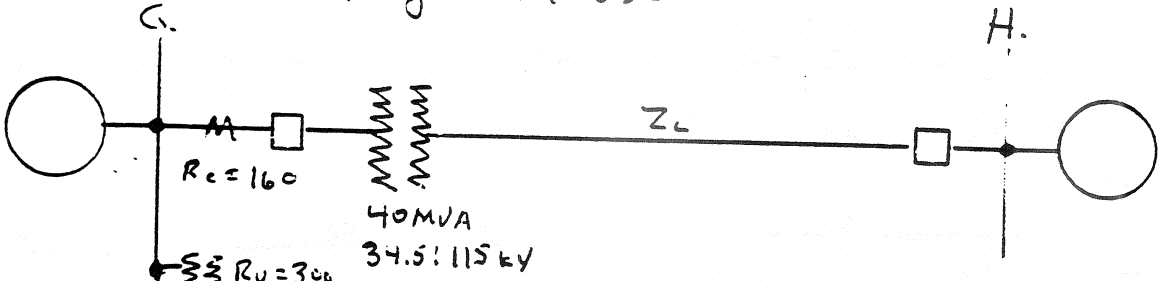
d) $= .16510 + j.09186 = .189\angle 29.09^\circ$ pu.



Zone 1 at H will operate as Z_{1H} is within the Zone 1 operating circle.

e) This internal fault can be cleared by high speed sequential operation: Zone 1 at H operates. When breaker at H opens the infeed is removed and zone 1 at G can operate as shown in part b diagram.

Problem 12.9 pages 29-530



$X_T = 8\%$ at 34.5 kV
$= 7.6\%$ at 38 kV
$= 8.5\%$ at 31 kV

a) $Z_L = 10\angle 80°$ ohms at 115 kV.

The line Z_L on the low voltage side appears as:

31 kV Tap	34.5 kV Tap	38 kV Tap
$Z_L = \left(\frac{31}{115}\right)^2 10\angle 80° = .727$ ohms.	$Z_L = \left(\frac{34.5}{115}\right)^2 10\angle 80° = .90\ \Omega$	$Z_L = \left(\frac{38}{115}\right)^2 10\angle 80 = 1.09\ \Omega$
$= .126 + j.716\ \Omega$	$= .156 + j.886\ \Omega$	$= .1894 + j1.075\ \Omega$
$Z_T = \frac{31^2 \times .085}{40} = j2.042\ \Omega$	$Z_T = \frac{34.5^2 \times .08}{40} = j2.381$	$Z_T = \frac{38^2 \times .076}{40} = j2.744$
$Z_{Total} = .126 + j2.758$	$Z_{Total} = .156 + j3.267$	$Z_{Total} = .19 + j3.82$
$= 2.76\angle 87.38°$ ohms from G.	$= 3.27\angle 87.27°$ ohms.	$= 3.82\angle 87.16°$ ohms

Set Zone for the lowest Total impedance which is when on the 31 kV Tap. Set $Z_1 = .99(j2.042) + .9(.126 + j.716) = j2.022 + .113 + j.644$

$= .113 + j2.666 = 2.67\angle 87.57°$ ohms.

Line Coverage on 31 kV Tap. Setting $.113 + j2.666$
less Z_T $j.2042$
$.113 + j.627 = .637\angle 79.78°$

b) 31 kV Tap; % Coverage $= \frac{.637}{.727} = 87.62\%$ of the line protected.

c) 34.5 kV Tap. Coverage is $.113 + j2.666$
less $Z_T = j2.381$
$.113 + j.285 = .3066\angle 68.37°$

% line protected $= \frac{.3066}{.90} = 34.07\%$.

38 kV Tap Coverage is $.113 + j2.666$
less Z_T $j2.744$ —— Relay does not reach through the Transformer. No line protection.

d) install high side (115 kV) voltage Transformers. * See pg 61

Prob. 12.10 Pages 529-530

$Z_{line} = 40\angle 80^\circ$ ohm at 115 kV. The line Z_L on the low voltage side appears as:

31 kV Tap

$Z_L = \left(\frac{31}{115}\right)^2 40\angle 80 = 2.907\ \Omega$

$= .505 + j2.862$ ohms.

$Z_T = \frac{31^2 \times .085}{40} = j2.042\ \Omega$

$Z_{Total} = .505 + j4.904$

$= 4.93\angle 84.12^\circ$ ohms.

34.5 kV Tap

$Z_L = \left(\frac{34.5}{115}\right)^2 40\angle 80 = 3.6\ \Omega$

$= .62 + j3.54$ ohms.

$Z_T = \frac{34.5^2 \times .08}{40} = j2.381$

$Z_{Total} = .62 + j5.92$

$= 5.95\angle 84.02^\circ$ ohms.

38 kV Tap

$Z_L = \left(\frac{38}{115}\right)^2 40\angle 80 = 4.37\ \Omega$

$= .758 + j4.301\ \Omega$

$Z_T = \frac{38^2 \times .076}{40} = j2.744\ \Omega$

$Z_{Total} = .758 + j7.045$

$= 7.08\angle 83.88^\circ$ ohms.

Set Zone for the lowest total impedance which is when using the 31 kV Tap.

Set Zone 1 for $.99(j2.042) + .99(.505 + j2.862)$

$= j2.022 + .454 + j2.576 = .454 + j4.598 = 4.62\angle 84.36^\circ$

31 kV Tap Line Coverage: set $.454 + j4.598$

less Z_T = $j2.042$

$.454 + j2.556 = 2.596\angle 79.93^\circ$ ohms.

% of line covered $\frac{2.596}{2.907} 100 = 89.30\%$ of line protected.

34.5 kV Tap Line Coverage: set $.454 + j4.598$

less Z_T = $-j2.381$

$.454 + j2.217 = 2.263\angle 78.42^\circ$ ohms.

% of line covered $= \frac{2.263}{3.6} 100 = 62.86\%$ of line protected

38 kV Tap Line Coverage: set $.454 + j4.598$

less Z_T $-j2.744$

$.454 + j1.854 = 1.91\angle 76.24^\circ$ ohms

% of line covered $= \frac{1.91}{4.37} 100 = 43.68\%$ of line protected

Protection of the line much better than for the 10 ohm line of Prob 12.9.
Best protection is to use 115 kV VTs. The low side CTs can be used if properly connected for the phase shift to provide transformer protection.

Problem 12.11 page 531-532.

Possible combinations for directional sensing for ground relaying:

a) $3I_0$ and $3V_0$; I_2 & V_2 using CTs(d) & VTs(h) at Bus G.
or using CTs (g) & VTs(i) at Bus H.

at G only; $3I_0$ from CTs (d) with $3I_0$ from CT(e) OR CT(f):

For Ground Relays at G.

	$3I_0$ - CTs(d) $R_c=100$	$3V_0$ - VTs (h) $R_v=600$	Gnd. neutral $3I_0$ - CT(e) $R_c=100$	Tertiary I_0 - CT(f) $R_c=40$
Fault 1 on line	$\frac{3\times2708}{100}=81.24$ a.	$\frac{23,206}{600}=38.67$ v.	$\frac{723}{100}=7.23$ a.	$\frac{554}{40}=13.85$ a.
Fault 2	$\frac{3\times1334}{100}=40.02$ a.	$\frac{11,428}{600}=19.05$ v.	$\frac{-1164}{100}=-11.64$ a.	$\frac{1857}{40}=46.43$ a.
Fault 3	$\frac{3\times160}{100}=4.80$ a.	$\frac{1375}{600}=2.29$ v.	$\frac{-138}{100}=-1.38$ a	$\frac{225}{40}=5.69$ a.

	I_2 - CTs(d) $R_c=100$	V_2 - VTs(h) $R_v=600$
Fault 1 on line	$\frac{2247}{100}=22.47$ a.	$\frac{16,046}{600}=26.74$ v.
Fault 2	$\frac{1150}{100}=11.50$ a	$\frac{8210}{600}=13.68$ v.
Fault 3	$\frac{673}{100}=6.73$ a.	$\frac{4806}{600}=8.01$ v.

For Ground Relays at H. Ratios of line CT's (g) at Bus H is not given but 250:5 ($R_c=50$) provides the same secondary current as those at G (d).

	$3I_0$ - CTs(g) $R_c=50$	$3V_0$ - VTs (i) $R_v=1200$	I_2 - CTs (g) $R_c=50$	V_2 - VTs (i) $R_v=1200$
Fault 3 on line	$\frac{3\times185}{50}=11.10$ a.	$\frac{52,888}{1200}=44.07$ v.	$\frac{1629}{50}=32.58$ a	$\frac{31,022}{1200}=25.85$ v.
Fault 2	$\frac{3\times137}{50}=8.22$ a.	$\frac{3912}{1200}=3.26$ v	$\frac{508}{50}=10.16$ a	$\frac{9,684}{1200}=8.07$ v
Fault 1.	$\frac{3\times74}{50}=4.44$ a	$\frac{2115}{1200}=1.76$ v.	$\frac{388}{50}=7.76$ a.	$\frac{7392}{1200}=6.16$ V.

Problem 12.11 Cont'd

b) On the basis that the various type of directional units have essentially equivalent sensitivities, negative sequence $V_2 I_2$ unit is recommended at both terminals. Adequate quantities are available especially for the remote faults where the zero sequence quantities tend to be low. However, all of the combinations except using the neutral current in the auto transformer could be used. Since the auto-transformer neutral current reverses for high side faults it cannot be used for directional sensing.

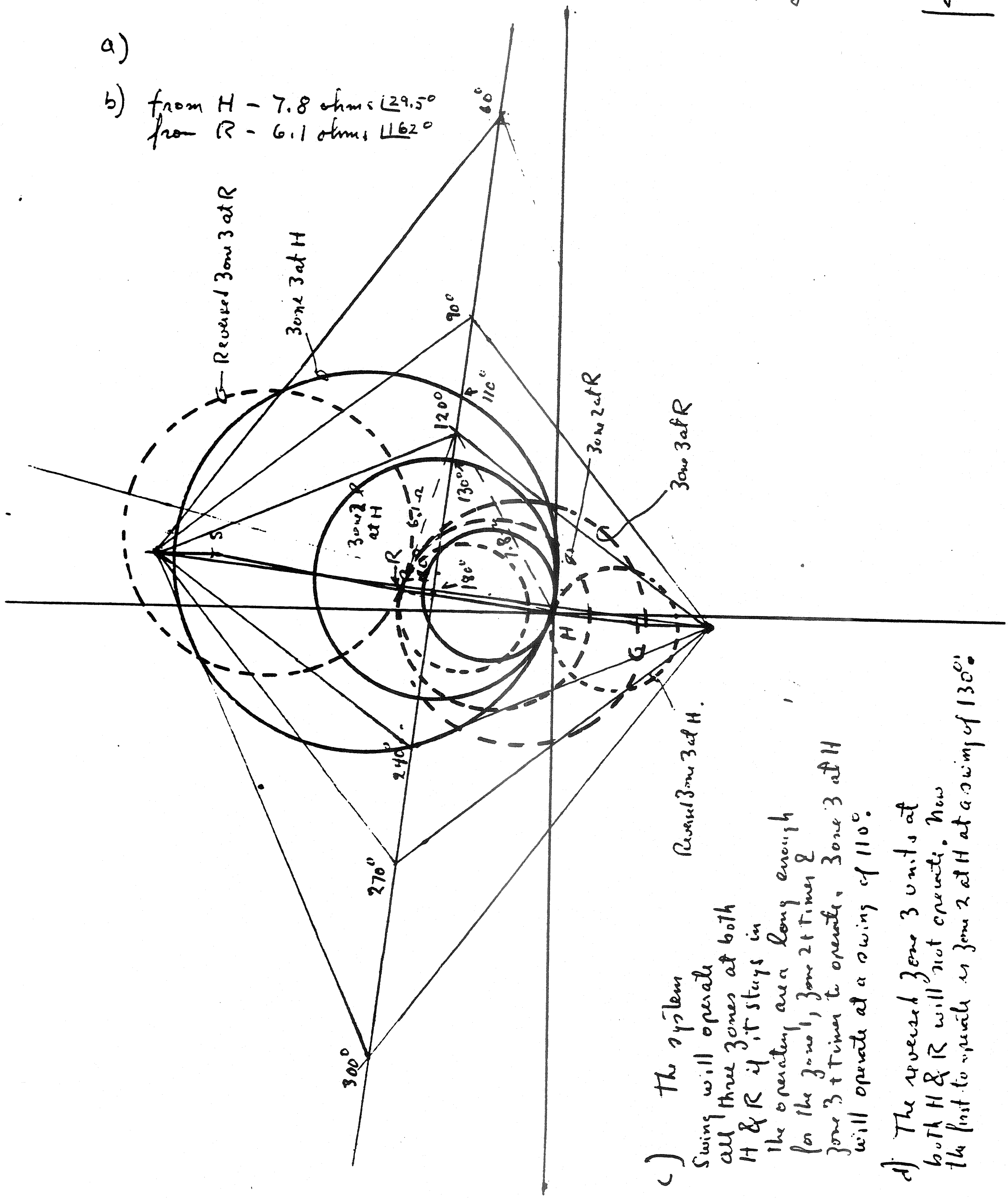
Problem 14.1 Page 532. NOTE Reference in problem should be To Problems 12.5 & 12.6 and not 12.4 & 12.3. use System Fig P12.5 Page 527.

See next page for chart, etc.

Problem 14.1 page 532 System of figure P12.5 page 527.

a)

b) from H – 7.8 ohms $\angle 29.5°$
from R – 6.1 ohms $\angle 162°$



c) The system
Swing will operate all three zones at both H & R if it stays in the operating area long enough for the zone 1, zone 2 timer & zone 3 timer to operate. Zone 3 at H will operate at a swing of 110°.

d) The reversed zone 3 units at both H & R will not operate. Now the first to operate is zone 2 at H at a swing of 130°.